Multiphonon variant of the coupled-channel method and strength functions of one-quasiparticle excitations

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A multiphonon variant of the coupled-channel method is formulated and applied to several problems. In contrast to the "few-phonon" approaches to describing the relaxation of one-quasiparticle excitations, the method approximately takes into account the coupling of one-quasiparticle states to multiphonon configurations, which is important for "soft" nuclei. The method is used to carry out a quantitative analysis of the one-quasiparticle and neutron strength functions, the neutron elastic-scattering radii, and also the strength function of the $E1$ giant resonance in spherical nuclei. The results are compared with the experimental data and with the results of other theoretical approaches.

INTRODUCTION

By now a considerable amount of experimental information has been accumulated on the properties of high-energy one-quasiparticle excitations in nuclei: there are data on the neutron and one-quasiparticle strength functions and on the parameters of the cross sections for low-energy neutron elastic scattering. The quantitative interpretation of these data requires the use of models which attempt to describe the relaxation of the one-quasiparticle degree of freedom in nuclei. One such model is the optical model of elastic scattering, in which a local energy-independent complex potential is used (see, for example, Ref. 1). In the optical model the relaxation of one-particle states of the continuum owing to their coupling to multiparticle configurations is taken into account phenomenologically and with averaging over the energy. The generalized optical model, or coupled-channel method (CCM, Ref. 2) explicitly takes into account the coupling of one-particle states to configurations containing phonons (as a rule, the latter are low-lying collective 2^+ states in even spherical nuclei), and the coupling to "noncollective" multiparticle configurations is taken into account using an optical potential which depends monotonically on the excitation energy. In practical applications of the CCM the relaxation of one-quasiparticle states of the discrete spectrum is not described, and, more importantly, the coupling of one-quasiparticle states only to configurations containing one or at most two phonons is taken into account. This latter statement is also true for the quasiparticle–phonon model (QPM, Ref. 3) and the approach described in Ref. 4. The restriction of the basis of phonon states is a consequence of the difficulty in practice of taking into account the coupling of quasiparticles to multiphonon configurations. However, it is necessary to take this coupling into account if the energy of the effective quasiparticle–phonon interaction is greater than the phonon energy. This necessity was demonstrated in Ref. 5 in connection with the theoretical analysis of the spectra of odd spherical nuclei.

In this review we formulate a model which is essentially a synthesis of the approaches used in Refs. 2 and 5 and in which, in addition to the coupling of one-quasiparticle states to multiparticle configurations in terms of the phenomenological optical potential, we approximately include the coupling of these states to multiphonon configurations. In this model, which can be termed a multiphonon variant of the coupled-channel method (MVCCM), we use the same phenomenological parameters as in the usual versions of the CCM. In addition to formulating the model, here we give the results of a quantitative MVCCM analysis of the one-quasiparticle and neutron strength functions and the radii of neutron elastic scattering by spherical nuclei. The MVCCM is used to solve the problem of the broadening of the electric giant dipole resonance in "soft" nuclei, since this problem reduces to a one-quasiparticle problem in some approximations.

In this review we give the results of studies carried out in recent years by B. B. Matveev, V. V. Samoĭlov, B. A. Tulupov, and the authors of the reviews of Refs. 6–9.

1. THE AVERAGED ONE-PARTICLE GREEN FUNCTION AND OBSERVABLES

Spectral expansion of the one-particle Green function

By definition, the one-particle Green function $G^\alpha(X,X')$ is[10,11]

$$G^\alpha(X,X') = -\mathrm{i}\langle 0|T\hat{\psi}_\alpha(X)\hat{\psi}_\alpha^+(X')|0\rangle, \qquad (1)$$

where $X = (\mathbf{r},t)\hat{\psi}_\alpha(X)$ is the Heisenberg annihilation operator of the neutron ($\alpha = n$) or proton ($\alpha = p$), T is the time-ordering operator, and the expectation value is taken in the ground state of the Hamiltonian $\hat{H}$ of the nucleus containing $A(Z,N)$ nucleons. In the absence of external fields the Green function (1) depends only on the time difference $t - t' = \tau$: $G(X,X') = G(\mathbf{r}\mathbf{r}',\tau)$. If we introduce a complete set of eigenfunctions of the Hamiltonian $\hat{H}$ for the nucleus containing $A+1$ or $A-1$ nucleons, $|s\rangle$ or $|\bar{s}\rangle$, the Green function (1) can be written as (here and below for simplicity we drop the isotopic index α, in which all the quantities considered are diagonal)

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$$G(\mathbf{r}\mathbf{r}',\tau)=\begin{cases}-\mathrm{i}\theta(\tau)\ \sum\limits_{s}\ \langle 0|\hat{\psi}(\mathbf{r})|s\rangle\langle s|\hat{\psi}^{+}(\mathbf{r}')|0\rangle\ \exp[-\mathrm{i}(E_s+\mu)\tau];\\ \mathrm{i}\theta(-\tau)\ \sum\limits_{\bar{s}}\ \langle 0|\hat{\psi}^{+}(\mathbf{r}')|\bar{s}\rangle\langle\bar{s}|\hat{\psi}(\mathbf{r})|0\rangle\ \exp[\mathrm{i}(E_{\bar{s}}+\mu)\tau].\end{cases} \tag{2}$$

Here E_s and $E_{\bar{s}}$ are the energies of the excited states of the nuclei containing $A+1$ and $A-1$ nucleons, respectively, μ is the chemical potential, $\theta(y)=1$ when $y>0$, and $\theta(y)=0$ when $y<0$. According to (2) the Fourier transform of the Green function $G(\mathbf{r}\mathbf{r}',\varepsilon)=\int d\tau G(\mathbf{r}\mathbf{r}',\tau)\exp[\mathrm{i}\varepsilon\tau]$ is

$$G(\mathbf{r}\mathbf{r}',\varepsilon)=\sum_{s}\frac{b_s(\mathbf{r}\mathbf{r}')}{\varepsilon-\mu-E_s+\mathrm{i}0}+\sum_{\bar{s}}\frac{b_{\bar{s}}(\mathbf{r}'\mathbf{r})}{\varepsilon-\mu+E_{\bar{s}}-\mathrm{i}0}$$
$$\equiv G^{(+)}+G^{(-)}, \tag{3}$$

where $b_s(\mathbf{r}\mathbf{r}')\equiv\langle 0|\hat{\psi}(\mathbf{r})|s\rangle\langle s|\hat{\psi}^{+}(\mathbf{r}')|0\rangle$, $\bar{b}_{\bar{s}}(\mathbf{r}'\mathbf{r})\equiv\langle 0|\hat{\psi}^{+}(\mathbf{r}')|\bar{s}\rangle\langle\bar{s}|\psi(\mathbf{r})|0\rangle$. In a system of noninteracting quasiparticles, when $\hat{H}\to\hat{H}_0=\Sigma_a H_0(a)$, where $H_0(\mathbf{r})$ is the one-particle Hamiltonian, according to (3) the Green function $G\to G_0$ can be written as

$$G_0(\mathbf{r}\mathbf{r}',\varepsilon)=\sum_{\lambda}\varphi_\lambda(\mathbf{r})\varphi_\lambda(\mathbf{r}')G_{0\lambda}(\varepsilon);$$

$$G_{0\lambda}(\varepsilon)=\frac{1-n_\lambda}{\varepsilon-\varepsilon_\lambda+\mathrm{i}0}+\frac{n_\lambda}{\varepsilon-\varepsilon_\lambda-\mathrm{i}0}, \tag{4}$$

where n_λ is the occupation number and $\varphi_\lambda(\mathbf{r})$ and ε_λ are the eigenfunctions and eigenvalues of the Hamiltonian $H_0(\mathbf{r})$: $(H_0(\mathbf{r})-\varepsilon_\lambda)\varphi_\lambda(\mathbf{r})=0$ [$\lambda=(njlm)$ when $\varepsilon_\lambda>0$, $\lambda=(\varepsilon jlm)$ when $\varepsilon>0$; in expressions in which the angular variables are eliminated we shall use λ to mean (njl) or (εjl)].

The averaged Green function and the one-quasiparticle strength functions

At sufficiently large excitation energies E_x ($E_x\sim B$, where B is the nucleon binding energy) it is interesting to describe the parameters of the cross sections for various nuclear reactions averaged over the energy interval $I\gg\rho_{s(\bar{s})}^{-1}$ [$\rho_{s(\bar{s})}$ is the density of poles of the Green function (3)]. In connection with this we introduce the averaged one-particle Green function $g(\mathbf{r}\mathbf{r}',\varepsilon)=\langle G(\mathbf{r}\mathbf{r}',\varepsilon)\rangle$, where the brackets $\langle\ldots\rangle$ denote the average over the energy interval I. The averaging procedure can be carried out using the analytic properties of the averaged function (see, for example, Ref. 12). For example, using (3) with accuracy $\sim I/|\varepsilon-\mu|$ we obtain

$$g(\mathbf{r}\mathbf{r}',\varepsilon)=\frac{1}{\pi}\int\frac{I}{(\varepsilon'-\varepsilon)^2+I^2}G(\mathbf{r}\mathbf{r}',\varepsilon')d\varepsilon'$$
$$=G(\mathbf{r}\mathbf{r}',\varepsilon+\mathrm{i}I\ \mathrm{Sgn}(\varepsilon-\mu)). \tag{5}$$

The quantities $b_s(\mathbf{r}=\mathbf{r}')$ and $\bar{b}_{\bar{s}}(\mathbf{r}'=\mathbf{r})$ have the meaning of the density of the probability for finding the one-particle or one-hole state, respectively, in the exact nuclear states, and the quantities $\rho_s\int\langle b_s(\mathbf{r}=\mathbf{r}')\rangle d\mathbf{r}$ and $\rho_{\bar{s}}\int\langle\bar{b}_{\bar{s}}(\mathbf{r}'=\mathbf{r})\rangle d\mathbf{r}$ are interpreted as the strength functions of the one-particle and one-hole states, respectively. Therefore, according to (3) the one-quasiparticle strength function is determined by the imaginary part of the averaged one-particle Green function:

$$S(\varepsilon)=-\frac{1}{\pi}\mathrm{Sgn}(\varepsilon-\mu)\int\mathrm{Im}\,g(\mathbf{r}=\mathbf{r}',\varepsilon)d\mathbf{r}. \tag{6}$$

For spherical nuclei the averaged Green function is conveniently represented as an expansion in spherical spinors $\Phi_{jlm}(\mathbf{r}/r')$:

$$g(\mathbf{r}\mathbf{r}',\varepsilon)=\frac{1}{rr'}\sum_{jlm}\Phi_{jlm}(\mathbf{r}/r)\Phi_{jlm}^{+}(\mathbf{r}'/r')g_{jl}(rr',\varepsilon). \tag{7}$$

Using this expansion, the strength function (6) can be written as

$$S(\varepsilon)=\sum_{jl}(2j+1)S_{jl}(\varepsilon);$$

$$S_{jl}(\varepsilon)=-\frac{1}{\pi}\mathrm{Sgn}(\varepsilon-\mu)\int_0^\infty\mathrm{Im}\,g_{jl}(r=r',\varepsilon)dr. \tag{8}$$

The strength functions $S_{jl}(\varepsilon)$ (8) are the subject of experimental study in one-nucleon transfer reactions.

The equation for the averaged Green function

The shell approach to the description of highly excited states of nuclei and nuclear reactions involving no more than one nucleon in the continuum corresponds to choosing the function $G_0(\mathbf{r}\mathbf{r}',\varepsilon)$ (4) as the zeroth approximation for the one-particle Green function. Then the exact Green function satisfies the Dyson equation

$$G(\mathbf{r}\mathbf{r}',\varepsilon)=G_0(\mathbf{r}\mathbf{r}',\varepsilon)+\int G_0(\mathbf{r}\mathbf{r}_1,\varepsilon)\Sigma(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$
$$\times G(\mathbf{r}_2\mathbf{r}',\varepsilon)d\mathbf{r}_1d\mathbf{r}_2. \tag{9}$$

Here Σ is the irreducible self-energy part describing the coupling of one-quasiparticle excitations to multi-quasiparticle ones with, by definition, $\Sigma(\varepsilon=\mu)=0$, and $\mathrm{Im}\,\Sigma(\varepsilon)$ changes sign at the point $\varepsilon=\mu$. In the range of excitation energies under consideration the quantity $\Sigma(\mathbf{r}\mathbf{r}',\varepsilon)$ is a strong function of the variable $\varepsilon-\mu$, as follows from (3) and (9). The equation for the averaged Green function (5) follows from the Dyson equation (9) if in this equation we make the replacement $\varepsilon\to\varepsilon+\mathrm{i}I\ \mathrm{Sgn}(\varepsilon-\mu)$:

$$g(\mathbf{rr}',\varepsilon)=g_0(\mathbf{rr}',\varepsilon)+\int g_0(\mathbf{rr}_1,\varepsilon)\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$

$$\times g(\mathbf{r}_2\mathbf{r}',\varepsilon)d\mathbf{r}_1d\mathbf{r}_2. \tag{10}$$

Here

$$\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)=\Sigma(\mathbf{r}_1\mathbf{r}_2,\varepsilon+iI\,\mathrm{Sgn}(\varepsilon-\mu))$$

$$=\langle\Sigma(\mathbf{r}_1\mathbf{r}_2,\varepsilon)\rangle \tag{11}$$

is the averaged irreducible self-energy part (the analytic properties of $G(\varepsilon)$ and $\Sigma(\varepsilon)$ coincide[10]), and $g_0(\mathbf{rr}',\varepsilon)$ is the Green function of the Schrödinger equation with the shell-model Hamiltonian:

$$(H_0(\mathbf{r})-\varepsilon)g_0(\mathbf{rr}',\varepsilon)=-\delta(\mathbf{r}-\mathbf{r}'), \tag{12}$$

as follows from (4). For positive nucleon energies the circuit of the poles in the spectral expansion of the function $g_0(\mathbf{rr}',\varepsilon)$ is dictated by the rule $\varepsilon\to\varepsilon+iI$. For negative energies it is necessary to choose an analytic continuation of this function such that its poles correspond to the nucleon bound states in accordance with (4).

We then use expansions of the form (7) for the functions g_0, g, and ΔH. Then according to (10) we obtain the equation for the radial part of the averaged Green function:

$$g_{jl}(rr',\varepsilon)=g_{0jl}(rr',\varepsilon)+\int g_{0jl}(rr_1,\varepsilon)$$

$$\times\Delta H_{jl}(r_1r_2,\varepsilon)g_{jl}(r_2r',\varepsilon)dr_1dr_2, \tag{13}$$

where according to (12)

$$(H_{0jl}(r)-\varepsilon)g_{0jl}(rr',\varepsilon)=-\delta(r-r'). \tag{14}$$

Here $H_{0jl}(r)$ is the radial part of the Hamiltonian $H_0(\mathbf{r})$:

$$H_{0jl}(r)=-\frac{\hbar^2}{2M}\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)+u(r)+(\mathbf{ls})_{jl}u_{s0}(r), \tag{15}$$

where $u(r)$ and $u_{so}(r)$ are, respectively, the central and spin–orbit parts of the shell potential. In connection with this we note that the eigenfunctions of the Hamiltonian $H_0(\mathbf{r})$ can be written as

$$\varphi_\lambda(\mathbf{r})=R_{0\varepsilon jl}(r)\Phi_{jlm}(\mathbf{r}/r);\ rR(r)=\chi(r), \tag{16}$$

where the radial wave functions $\chi_{0\varepsilon jl}(r)$ satisfy the equation

$$(H_{0jl}(r)-\varepsilon)\chi_{0jl}(r)=0 \tag{17}$$

and the condition $\chi_{0\varepsilon jl}(0)=0$.

Equations (10) and (13) therefore correspond to the problem of the motion of a nucleon in a nonlocal, energy-dependent, complex potential. It is necessary to parametrize $\Delta H(\mathbf{rr}',\varepsilon)$ using the minimum number of phenomenological parameters on the basis of a "semimicroscopic" model.

The averaged amplitude and wave function for the elastic scattering of a nucleon by a nucleus

The amplitude $f_{0\mu\mu'}(\mathbf{kk}'/k^2)$ and the wave functions for nucleon scattering with the shell potential $\psi^{\pm}_{0\mathbf{k}\mu}(\mathbf{r})$ having the asymptotic form of a plane wave plus outgoing wave can be written as partial-wave series:

$$f_{0\mu\mu'}\left(\frac{\mathbf{kk}'}{k^2}\right)=\frac{2\pi}{ik}\sum_{jlm}\left(\Phi^+_{jlm}\left(\frac{\mathbf{k}'}{k}\right)\chi_{\mu'}\right)^*\left(\Phi^+_{jlm}\left(\frac{\mathbf{k}}{k}\right)\chi_{\mu}\right)$$

$$\times(S_{0jl}-1); \tag{18}$$

$$\psi^{(\pm)}_{0\mathbf{k}\mu}(\mathbf{r})=4\pi\sum_{jlm}i^l\left(\Phi^+_{jlm}\left(\frac{\mathbf{k}}{k}\right)\chi_{\mu}\right)\Phi_{jlm}\left(\frac{\mathbf{r}}{r}\right)R^{(\pm)}_{0kjl}(r). \tag{19}$$

Here χ_μ are the spin wave functions; $k=(2M\varepsilon/\hbar^2)^{1/2}$ is the nucleon wave vector; $S_{0jl}=\exp[2i\delta_{0jl}]$, where δ_{0jl} are the scattering phase shifts;

$$rR^{(\pm)}_{0kjl}(r)=\chi^{(\pm)}_{0kjl}(r);\quad \chi^{(-1)}_{0kjl}=\chi^{(+)*}_{0kjl} \tag{20}$$

are the radial wave functions satisfying Eq. (17) and the condition $\chi(0)=0$. In the limit $r\gg R$ (R is the radius of the nuclear part of the shell potential) these functions have the form

$$\chi^{(+)}_{0kjl}(r)\to(2ik)^{-1}[-u^{(-)}_l(kr)+S_{0jl}(k)u^{(+)}_l(kr)]. \tag{21}$$

In the case of neutrons $u^{(\pm)}_l(x)=\pm ixh^{(1,2)}_l(x)$, where $h^{(1,2)}_l(x)$ are the spherical Hankel functions of the first and second kind. In the case of protons $u^{(\pm)}_l(x)=G_l(x)\pm iF_l(x)$, where $F_l(x)$ and $G_l(x)$ are the corresponding Coulomb functions and δ_{0jl} in (21) is in this case the nuclear part of the total phase shift. The radial wave functions of the continuum normalized to a δ function in the energy $\chi^{(\pm)}_{0\varepsilon jl}(r)$ are related to the wave functions (20) as

$$\chi^{(\pm)}_{0\varepsilon jl}(r)=(2Mk/\pi\hbar^2)^{1/2}\chi^{(\pm)}_{0kjl}(r). \tag{22}$$

The elastic scattering of a nucleon by a nucleus is accompanied by the excitation of compound states. The corresponding addition to the amplitude of the potential scattering (18) $\delta f_{\mu'\mu}$ ($f_{\mu'\mu}=f_{0\mu'\mu}+\delta f_{\mu'\mu}$) is determined by the irreducible self-energy part $\Sigma(\mathbf{rr}',\varepsilon)$, since the latter is interpreted as the part of the nucleon–nucleus interaction Hamiltonian related to the excitation of compound states. From this statement it follows that

$$\delta f_{\mu'\mu}\left(\frac{\mathbf{kk}'}{k^2},\varepsilon\right)=-\frac{M}{2\pi\hbar^2}\int\psi^{(-)*}_{0k'\mu'}(\mathbf{r}_1)\Sigma(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$

$$\times\Psi^{(+)}_{\mathbf{k}\mu}(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2. \tag{23}$$

The "resonance" wave functions of the scattering problem $\Psi^{(+)}_{\mathbf{k}\mu}(\mathbf{r},\varepsilon)$ satisfy the equation

$$\Psi_{k\mu}^{(+)}(\mathbf{r},\varepsilon)=\psi_{0k\mu}^{(+)}(\mathbf{r})+\int g_0(\mathbf{r}\mathbf{r}_1,\varepsilon)\Sigma(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$

$$\times\Psi_{k\mu}^{(+)}(\mathbf{r}_2,\varepsilon)d\mathbf{r}_1d\mathbf{r}_2. \quad (24)$$

Averaging Eqs. (23) and (24) over energy by the replacement $\varepsilon\to\varepsilon+iI$, we obtain expressions relating the quantities $\langle\delta f\rangle$, $\langle\Sigma\rangle$, and the averaged wave function of the scattering problem $\psi^{(+)}=\Psi^{(+)}(\varepsilon+iI)$:

$$\delta f_{\mu'\mu}\left(\frac{\mathbf{k}\mathbf{k}'}{k^2},\varepsilon\right)=-\frac{M}{2\pi\hbar^2}\int\psi_{0k'\mu'}^{(-)*}(\mathbf{r}_1)\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$

$$\times\psi_{k\mu}^{(+)}(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2, \quad (25)$$

where

$$\psi_{k\mu}^{(+)}(\mathbf{r})=\psi_{0k\mu}^{(+)}(\mathbf{r})+\int g_0(\mathbf{r}\mathbf{r}_1,\varepsilon)\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$$

$$\times\psi_{k\mu}^{(+)}(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2. \quad (26)$$

Transforming in the last equation to the partial-wave expansion and using (7) and (19), we obtain the corresponding equations for the radial wave functions:

$$\chi_{kjl}^{(+)}(r)=\chi_{0kjl}^{(+)}(r)+\int g_{0jl}(rr_1,\varepsilon)\Delta H_{jl}(r_1r_2,\varepsilon)$$

$$\times\chi_{kjl}^{(+)}(r_2)dr_1dr_2. \quad (27)$$

In order to obtain an explicit expression for the averaged scattering amplitude we turn to the well known representation of the Green function $g_{0jl}(rr',\varepsilon)$ following from Eq. (12) (see, for example, Ref. 13):

$$g_{0jl}(rr',\varepsilon)=-\frac{2M}{\hbar^2}v_{0kjl}(r_>)\chi_{0kjl}^{(+)}(r_<);\quad \begin{matrix} r_>=\max[rr'], \\ r_<=\min[rr']. \end{matrix} \quad (28)$$

Here $\chi_{0kjl}^{(+)}$ and v_{0kjl} are the solutions of this equation which are regular and irregular at the origin and, respectively, have the asymptotic form (21) and

$$v_{0kjl}(r)\to u_l^{(+)}(kr). \quad (29)$$

The notation is the same as in (21). Taking the limit $r\to\infty$ in Eq. (27) and taking into account (28), (29), and (22), we find a relation between the elements of the averaged scattering matrix and the S matrix of potential scattering:

$$\langle S_{jl}\rangle=S_{0jl}-2\pi i\int\chi_{0\varepsilon jl}^{(+)}(r)\Delta H_{jl}(rr',\varepsilon)$$

$$\times\chi_{\varepsilon jl}^{(+)}(r')drdr'. \quad (30)$$

The averaged elastic scattering amplitude is determined by the elements $\langle S_{jl}\rangle$, just as the amplitude for potential scattering is determined by the elements S_{0jl} in (18). This statement follows from (18), (19), (25), (26), and (30).

Therefore, as in the case of the averaged Green function, the expressions obtained for the averaged scattering amplitude (the averaged S matrix) and the averaged wave function for nucleon scattering with the excitation of compound states correspond to the problem of nucleon scattering on a nonlocal, energy-dependent complex potential. The latter problem can be solved if the form of the function $\Delta H(rr',\varepsilon)$ is given.

The neutron strength functions

For positive nucleon energies the literal application of (8) for the one-quasiparticle strength function is meaningless. In fact, when $\varepsilon>0$ the integral in (8) diverges at the upper limit [this can be verified using an expression of the form (28) for $g_{jl}(r=r',\varepsilon>0)$], which reflects the possibility of particles going to infinity. Therefore, for positive energies we need a different method of analyzing the parameters of the compound states. In problems of a continuous spectrum with a single nucleon in the continuum the averaged parameters of the compound states can be related to the averaged amplitudes and cross sections of the corresponding reactions. In this section we shall consider the parametrization of the S-matrix element corresponding to neutron elastic scattering S_{nn} and then average this quantity over the energy.

Assuming that the compound resonances correspond to simple poles of the scattering matrix, in the energy range near one of the nonoverlapping resonances (with energy ε_c) the quantity S_{nn} can be written as

$$S_{nn}(\varepsilon)=e^{2i\xi_n}\left(1-\frac{i\gamma_{nc}}{\varepsilon-\varepsilon_c+i\gamma_c/2}\right). \quad (31)$$

Here γ_{nc} and γ_c are the neutron width and the total width of the compound resonances, ξ_n is the phase shift, which varies smoothly in the energy range $I\gg\rho^{-1}$, where, as before, ρ is the density of compound states with given values of the spin and parity. The quantity S_{nn} (31) averaged over the energy interval I,

$$\langle S_{nn}\rangle\equiv e^{2i\delta_n}=e^{2i\xi_n}(1-\pi S_n), \quad (32)$$

determines the neutron strength function $S_n=\rho\gamma_n$ (γ_n is the averaged neutron width of the compound resonances) and the elastic-scattering radius $R_n\sim\xi_n$. Later we will use Eq. (32) to calculate the neutron strength function and the neutron elastic-scattering radius.

2. TRANSITION TO THE OPTICAL MODEL AND THE COUPLED-CHANNEL METHOD

Transition to the optical model

It follows from the discussion of Sec. 1 that the description of the relaxation of one-quasiparticle excitations using the shell-model approach and energy averaging requires knowledge of the average of the irreducible self-energy part (11). It is not possible to realize a systematic "microscopic" approach for calculating $\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$. A considerably more "economic" approach is that of "semimicroscopic" models, in which the quantity ΔH is, at least partially, not calculated, but is parametrized using certain physical assumptions. Ignoring for the time being

the existence of phonons (we denote the corresponding part of Σ by Σ^m), we give several diagrams corresponding to the lowest orders of perturbation theory in the quasiparticle interaction which contribute to Σ^m:

Here the thin lines denote the Green functions G_0 (4), and the points are the matrix elements of the quasiparticle interaction. According to these diagrams, the first step in analyzing the "decay" of a one-quasiparticle state (the first step in solving the corresponding diagonalization problem) is the inclusion of its coupling to three-quasiparticle configurations (incoming states[14]). If (i) most of the incoming states give comparable contributions to Σ^m, so that a large momentum (on the order of the Fermi momentum) is transferred to the incoming states in the "decay" process, and (ii) the fragmentation widths (the widths for "decay" into configurations more complicated than three-quasiparticle ones) of the incoming states overlap, then it can be expected that the quantity $\Delta H(\mathbf{r}_1\mathbf{r}_2,\varepsilon)$ contains a term representing a local potential with monotonic energy dependence:

$$\Delta H(\mathbf{rr},\varepsilon)\to\Delta H^m(\mathbf{rr}',\varepsilon)=\Delta H^m(r,\varepsilon)\delta(\mathbf{r}-\mathbf{r}'); \quad (33)$$

$$\Delta H^m(r,\varepsilon)=\Delta(r,\varepsilon)-iw(r\mid\varepsilon-\mu\mid)\,\mathrm{Sgn}(\varepsilon-\mu), \quad (34)$$

where $\Delta H^m(\varepsilon\to\mu)\to 0$ and $w>0$. In the approximation (33), (34) the averaged Green functions $g(\mathbf{rr}',\varepsilon)$, $g_{jl}(rr',\varepsilon)$, like the averaged wave functions of the scattering problem $\psi^{(+)}_{\mathbf{k}\mu}(\mathbf{r})$, $\chi^{(+)}_{kjl}(r)$, according to (10), (13) and (26), (27) coincide with the corresponding functions of the optical model of elastic scattering with a local potential and the Hamiltonian

$$\left.\begin{aligned} &H^{\mathrm{opt}}(\mathbf{r},\varepsilon)=H_0(\mathbf{r})+\Delta H^m(r,\varepsilon);\\ &H^{\mathrm{opt}}_{jl}(r,\varepsilon)=H_{0jl}(r)+\Delta H^m(r,\varepsilon). \end{aligned}\right\} \quad (35)$$

Therefore,

$$\left.\begin{aligned} &g(\mathbf{rr}',\varepsilon)\to g^{\mathrm{opt}}(\mathbf{rr}',\varepsilon);\\ &(H^{\mathrm{opt}}(\mathbf{r},\varepsilon)-\varepsilon)g^{\mathrm{opt}}(\mathbf{rr}',\varepsilon)=-\delta(\mathbf{r}-\mathbf{r}');\\ &g_{jl}(rr',\varepsilon)\to g^{\mathrm{opt}}_{jl}(rr',\varepsilon);\\ &(H^{\mathrm{opt}}_{jl}(r,\varepsilon)-\varepsilon)g^{\mathrm{opt}}_{jl}(rr',\varepsilon)=-\delta(r-r') \end{aligned}\right\} \quad (36)$$

and

$$\left.\begin{aligned} &\psi^{(+)}_{\mathbf{k}\mu}(\mathbf{r})\to\psi^{(+)\,\mathrm{opt}}_{\mathbf{k}\mu}(\mathbf{r});\\ &(H^{\mathrm{opt}}(\mathbf{r},\varepsilon)-\varepsilon)\psi^{(+)\,\mathrm{opt}}_{\mathbf{k}\mu}(\mathbf{r})=0;\\ &\chi^{(+)}_{kjl}(r)\to\chi^{(+)\,\mathrm{opt}}_{kjl}(r);\\ &(H^{\mathrm{opt}}_{jl}(r,\varepsilon)-\varepsilon)\chi^{(+)\,\mathrm{opt}}_{kjl}(r)=0. \end{aligned}\right\} \quad (37)$$

In the optical model of elastic scattering only Eqs. (37) are used. In particular, the last of these determines the phase shift for scattering by the optical potential and, consequently, the averaged S matrix and scattering amplitude in the optical-model approximation:

$$\langle f\rangle\to f^{\mathrm{opt}};\quad \langle S\rangle\to S^{\mathrm{opt}}=\exp[2i\delta^{\mathrm{opt}}_{jl}];\quad \delta^{\mathrm{opt}}_{jl}=\xi^{\mathrm{opt}}_{jl}+i\eta^{\mathrm{opt}}_{jl}, \quad (38)$$

where $\eta^{\mathrm{opt}}_{jl}>0$ owing to (34).

For nucleon energies $\varepsilon>0$, Eq. (36) can be used to write the optical-model Green function in a form similar to (28):

$$g^{\mathrm{opt}}_{jl}(rr',\varepsilon)=-(2M/\hbar^2)v^{\mathrm{opt}}_{kjl}(r_>)\chi^{(+)\,\mathrm{opt}}_{kjl}(r_<), \quad (39)$$

where the functions $v^{\mathrm{opt}}_{kjl}(r)$ and $\chi^{(+)\mathrm{opt}}_{kjl}(r)$ satisfy the second of Eqs. (37) and the boundary conditions $\chi^{(+)\mathrm{opt}}_{klj}(0)=0$:

$$\left.\begin{aligned} &\chi^{(+)\,\mathrm{opt}}_{kjl}(r)\underset{r\gg R}{\to}(2ik)^{-1}[-u^{(-)}_l(kr)\\ &\qquad\qquad +S^{\mathrm{opt}}_{jl}(k)u^{(+)}_l(kr)],\\ &v^{\mathrm{opt}}_{kjl}(r)\underset{r\gg R}{\to}u^{(+)}_l(kr)\quad(\varepsilon>0). \end{aligned}\right\} \quad (40)$$

The functions $u^{(\pm)}_l(x)$ were found above [below Eq. (21)]. For negative nucleon energies ($\varepsilon=-\hbar^2\varkappa^2/2M$) the replacement $k\to i\varkappa$ gives, instead of (40),

$$\left.\begin{aligned} &\chi^{(+)\,\mathrm{opt}}_{\varkappa jl}(r)\underset{r\gg R}{\to}(2\varkappa)^{-1}[w^{(+)}_l(\varkappa r)\\ &\qquad\qquad -B_{jl}(\varkappa)w^{(-)}_l(\varkappa r)],\\ &v_{\varkappa jl}(r)\underset{r\gg R}{\to}w^{(-)}_l(\varkappa r), \end{aligned}\right\} \quad (41)$$

where $w^{(\pm)}_l(y)=u^{(\mp)}_l(iy)$. In the case of neutrons the functions $w^{(\pm)}_l(y)$ are expressed in terms of the modified Bessel functions. In the case of protons they are given in terms of Whittaker functions with doubled argument.

The one-quasiparticle and neutron strength functions in the optical-model approximation

In the optical-model approximation, when $\Delta H\to\Delta H^m$, $g\to g^{\mathrm{opt}}$, $\chi^{(+)}\to\chi^{(+)\mathrm{opt}}$, it is possible to calculate the strength functions $S_{jl}(\varepsilon)$ and S_n according to (8) and (30), (32), respectively, if the parameters of the optical addition to the shell-model potential (34) are given. Calculations of this type for neutron strength functions were carried out long ago (see, for example, Refs. 1 and 15), and relatively recently for $S_{jl}(\varepsilon)$ (Ref. 16). Examples of such calculations are given in Secs. 4 and 5. Here we discuss the quantitative analysis of the conclusions of the optical-model approach.

In an energy interval near a discrete (or quasidiscrete) level, i.e., in an interval $|\varepsilon-\varepsilon_\lambda|\ll D_{(\lambda)}$ [$D_{(\lambda)}$ is the energy interval between adjacent one-particle levels with identical values of the angular momentum and parity], the results of the optical-model approach can be represented analytically if we use the approximate expressions for the Green function $g_{0\lambda}(rr',\varepsilon<0)$ according to (4),

$$g_{0\lambda}(rr',\varepsilon<0)\simeq(\varepsilon-\varepsilon_\lambda)^{-1}\chi_\lambda(r)\chi_\lambda(r'), \quad (42)$$

and also for the Green function $g_{0\lambda}(rr',\varepsilon>0)$ corresponding to the S-matrix element $S_{0\lambda\lambda}$ and the continuum wave

function $\chi_{0\lambda}^{(+)}(r)$ for the scattering of a nucleon by the shell-model potential (see, for example, Ref. 13):

$$g_{0\lambda}(rr',\varepsilon>0)\simeq\left[\varepsilon-\varepsilon_\lambda+\frac{\mathrm{i}}{2}\Gamma_\lambda^\uparrow\right]^{-1}\chi_\lambda^{(0)}(r)\chi_\lambda^{(0)}(r'); \tag{43}$$

$$S_{0\lambda\lambda}(\varepsilon)\simeq e^{2i\delta_{0\lambda}^{(0)}}\left[1-\frac{i\Gamma_\lambda^\uparrow}{\varepsilon-\varepsilon_\lambda+\frac{\mathrm{i}}{2}\Gamma_\lambda^\uparrow}\right];$$

$$\chi_{0\lambda}^{(+)}(r)=\left[\frac{\Gamma_\lambda^\uparrow}{2\pi}\right]^{1/2}\frac{\chi_\lambda^{(0)}(r)}{\varepsilon-\varepsilon_\lambda+\frac{\mathrm{i}}{2}\Gamma_\lambda^\uparrow}. \tag{44}$$

Here $\Gamma_\lambda^\uparrow$ is the width of the quasidiscrete level due to the existence of the one-particle continuum [$\Gamma_\lambda^\uparrow \ll D_{(\lambda)}$]; $\delta_{0\lambda}^{(0)}$ is the nonresonance part of the phase shift of potential scattering, and $\chi_\lambda^{(0)}(r)$ is the corresponding solution of Eq. (17) normalized to unity in the nuclear volume. Solution of Eqs. (36) for $g_\lambda^{\mathrm{opt}}(rr',\varepsilon)$ and (37) for $\chi_\lambda^{(+)\,\mathrm{opt}}(r)$ followed by calculation of $S_{\lambda\lambda}^{\mathrm{opt}}(\varepsilon)$ according to (30) leads to expressions of the form (42)–(44) with the replacement

$$\varepsilon_\lambda\to\tilde{\varepsilon}_\lambda-\frac{\mathrm{i}}{2}\tilde{\Gamma}_\lambda^\downarrow;\quad \tilde{\varepsilon}_\lambda=\varepsilon_\lambda+\Delta_\lambda;\quad \tilde{\Gamma}_\lambda^\downarrow=\Gamma_\lambda^\downarrow\,\mathrm{Sgn}(\varepsilon-\mu);$$

$$\Delta_\lambda-\frac{\mathrm{i}}{2}\tilde{\Gamma}_\lambda^\downarrow=\begin{cases}\int\chi_\lambda^2(r)\Delta H^m(r,\varepsilon_\lambda)dr & (\varepsilon_\lambda<0);\\ \int\chi_\lambda^{(0)^2}(r)\Delta H^m(r,\varepsilon_\lambda)dr & (\varepsilon_\lambda>0).\end{cases} \tag{45}$$

Further calculation of the one-quasiparticle strength function using (8) and the neutron strength function using (32) leads to the following expressions for these quantities ($\Gamma_\lambda^\uparrow\gg\Gamma_\lambda^\downarrow$):

$$S_\lambda(\varepsilon)=\frac{1}{2\pi}\frac{\Gamma_\lambda^\downarrow}{(\varepsilon-\tilde{\varepsilon}_\lambda)^2+\frac{1}{4}(\Gamma_\lambda^\downarrow)^2};\quad S_{n\lambda}=\Gamma_\lambda^\uparrow S_\lambda(\varepsilon). \tag{46}$$

We note that $\int S_\lambda(\varepsilon)d\varepsilon=1$. Equations (46) admit a clear interpretation based on the definitions of the strength functions:

$$\gamma_n=\Gamma_\lambda^\uparrow\langle b_s^\lambda\rangle;\quad \langle b_s^\lambda\rangle=\frac{1}{2\pi\rho_s}\frac{\Gamma_\lambda^\downarrow}{(\varepsilon-\tilde{\varepsilon}_\lambda)^2+\frac{1}{4}(\Gamma_\lambda^\downarrow)^2}; \tag{47}$$

$$b_s^\lambda=\int b_s(\mathbf{rr}')\varphi_\lambda^*(\mathbf{r})\varphi_\lambda(\mathbf{r}')d\mathbf{r}d\mathbf{r}',$$

i.e., the averaged neutron width of the compound resonances is equal to the product of the one-particle width for decay into the continuum and the averaged probability of finding a one-quasiparticle state in the exact nuclear states. According to (45)–(47), the quantities $\Gamma_\lambda^\downarrow$ and Δ_λ are, respectively, interpreted as the (fragmentation) width and the (fragmentation) shift of the one-quasiparticle state owing to the coupling of this state to multiparticle configurations.

The derivation of the approximate expression (46) for $S_\lambda(\varepsilon)$ based on the approximate representation (43) shows that it is necessary to modify Eq. (8) for the calculation of the strength functions of subbarrier one-particle states. In this case in the integral (8), which formally diverges at the upper limit, the integration region must be limited by the value of r_λ for which the condition $\int S_\lambda(\varepsilon)d\varepsilon=1$ is satisfied. Naturally, the radius r_λ is located in the subbarrier region. This procedure is actually equivalent to the method of normalizing the wave functions $\chi_\lambda^{(0)}(r)$ in (43).

The approximate solutions for the optical-model functions which lead to the approximate expressions (46) for the strength functions, are valid when

$$\left|\Delta_\lambda-\frac{\mathrm{i}}{2}\Gamma_\lambda^\downarrow\right|\ll D_{(\lambda)}, \tag{48}$$

which is the condition for the shells not to vanish and is therefore the criterion for the shell approach to be applicable. With the accuracy (48) we can neglect the nonorthogonality of the optical-model wave functions when calculating those matrix elements which are nonzero and for $\Delta H^m=0$.

Transition to the optical model with coupling between channels

At certain excitation energies it can turn out that one or several incoming states can, in spite of their small statistical weight, give a contribution comparable with $\Delta H^m(\mathbf{rr}',\varepsilon)$ to the averaged irreducible self-energy part $\Delta H(\mathbf{rr}',\varepsilon)$ (11). In this case the quantities Σ in (9) and ΔH in (10) can be written as sums

$$\Sigma=\Sigma^m+\Sigma^p,\quad \Delta H=\Delta H^m+\Delta H^p, \tag{49}$$

where $\Delta H^p=\langle\Sigma^p\rangle$ represents, in general, a potential which is nonlocal and has nonmonotonic energy dependence. The specific form of ΔH^p depends on the nature of these incoming states and will be studied below for quasiparticle–phonon configurations.

After substituting the self-energy part Σ (49) into the Dyson equation (9) and averaging over the energy we arrive at the equation for the averaged Green function:

$$g(\mathbf{rr}',\varepsilon)=g^{\mathrm{opt}}(\mathbf{rr}',\varepsilon)+\int g^{\mathrm{opt}}(\mathbf{rr}_1,\varepsilon)\Delta H^p(\mathbf{r}_1\mathbf{r}_2,\varepsilon)\times g(\mathbf{r}_2\mathbf{r}',\varepsilon)d\mathbf{r}_1d\mathbf{r}_2, \tag{50}$$

in which the input is not the shell-model function, but the Green function of the optical model. A similar representation can be obtained for the equation for the averaged wave function of the scattering problem:

$$\psi_{k\mu}^{(+)}(\mathbf{r})=\psi_{k\mu}^{(+)\,\mathrm{opt}}(\mathbf{r})+\int g^{\mathrm{opt}}(\mathbf{rr}_1,\varepsilon)\Delta H^p(\mathbf{r}_1\mathbf{r}_2,\varepsilon)\times\psi_{k\mu}^{(+)}(\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2. \tag{51}$$

After eliminating the angular variables in (50) and (51) we arrive at equations for the corresponding radial functions:

$$g_{jl}(rr',\varepsilon)=g_{jl}^{\rm opt}(rr',\varepsilon)+\int g_{jl}^{\rm opt}(rr_1,\varepsilon)\Delta H_{jl}^{p}(r_1r_2,\varepsilon)\times g_{jl}(r_2r',\varepsilon)dr_1dr_2; \quad (52)$$

$$\chi_{kjl}^{(+)}(r)=\chi_{kjl}^{(+)\,\rm opt}(r)+\int g_{jl}^{\rm opt}(rr_1,\varepsilon)\Delta H_{jl}^{p}(r_1r_2,\varepsilon)\times\chi_{kjl}^{(+)}(r_2)dr_1dr_2. \quad (53)$$

Equation (53) and the representation (39) lead to an expression for the elements of the averaged S matrix of nucleon elastic scattering analogous to (30):

$$\langle S_{jl}(\varepsilon)\rangle=S_{jl}^{\rm opt}(\varepsilon)-2\pi i\int\chi_{\varepsilon jl}^{(+)\,\rm opt}(r_1)\Delta H_{jl}^{p}(r_1r_2,\varepsilon)\times\chi_{\varepsilon jl}^{(+)}(r_2)dr_1dr_2. \quad (54)$$

Here the functions $\chi_{\varepsilon jl}^{(+)}$ are related to the functions $\chi_{kjl}^{(+)}$ by an expression of the form (22).

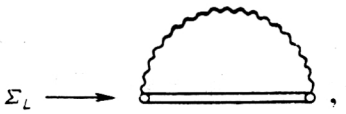
In spherical nuclei the initial states whose contribution to Σ^p must be taken into account exactly are "quasiparticle–phonon" configurations if there is strong coupling between the quasiparticles and the phonons, when this coupling leads to a significant rearrangement of the one-quasiparticle spectrum. The phonons are mainly low-lying collective 2^+ states in spherical nuclei. Let ω_L and L be the phonon energy and angular momentum, respectively, and $V_L(\mathbf{r})$ be the field acting on the quasiparticle in the phonon production process. In the "macroscopic" approach, where the phonon corresponds to nuclear-surface oscillation quanta, the field $V_L(\mathbf{r})$ is proportional to the dynamical-deformation parameters of the nucleus:[17]

$$V_L(\mathbf{r})=V(r)\sum_{\mu}\alpha_{L\mu}Y_{L\mu}(\mathbf{r}/r);\quad V(r)=-R\frac{du}{dr}, \quad (55)$$

where $u(r)$ is the mean field of the shell model. The parameters $\beta_L=(\Sigma_\mu|\alpha_{L\mu}|^2)^{1/2}$ can be found, for example, from the data on the Coulomb excitation of one-phonon states in the neighboring even nucleus. In "microscopic" approaches the field $V_L(\mathbf{r})$ is completely determined by the transition with excitation of a one-phonon state and by the effective quasiparticle interaction in the particle–hole channel.[18–20]

The one-phonon variant of the coupled-channel method (CCM)

In lowest order in the interaction (55) the contribution to the irreducible self-energy part $\Sigma^p=\Sigma_L\Sigma_L$ is described by the following graph:



where the double line corresponds to the Green function G^m satisfying the equation

$$G^m=G_0+G_0\Sigma^mG^m, \quad (56)$$

the wavy line corresponds to the phonon Green function $D_L(\varepsilon)$ (Ref. 11),

$$D_L(\varepsilon)=\frac{1}{\varepsilon-\omega_L+i\delta}-\frac{1}{\varepsilon+\omega_L-i\delta}, \quad (57)$$

and the circles correspond to the matrix element (in the space of phonon occupation numbers), for which we retain the notation used for the interaction (55). The above diagram corresponds to the following analytic expression:[11]

$$\Sigma_L(\mathbf{r}\mathbf{r}',\varepsilon)=-V_L^*(\mathbf{r})V_L(\mathbf{r}')\times\int G^m(\mathbf{r}\mathbf{r}',\varepsilon')D_L(\varepsilon-\varepsilon')d\varepsilon'/2\pi i. \quad (58)$$

Carrying out the integration in this expression, taking into account (3) and (57), we obtain two equivalent expressions for Σ_L:

$$\Sigma_L(\mathbf{r}\mathbf{r}',\varepsilon)=V_L^*(\mathbf{r})V_L(\mathbf{r}')\begin{cases}G^m(\mathbf{r}\mathbf{r}',\varepsilon-\omega_L)+[G^{m(-)}(\mathbf{r}\mathbf{r}',\varepsilon+\omega_L)-G^{m(-)}(\mathbf{r}\mathbf{r}',\varepsilon-\omega_L)];\\ G^m(\mathbf{r}\mathbf{r}',\varepsilon+\omega_L)+[G^{m(+)}(\mathbf{r}\mathbf{r}',\varepsilon-\omega_L)-G^{m(+)}(\mathbf{r}\mathbf{r}',\varepsilon+\omega_L)].\end{cases} \quad (59)$$

In the first expression the quantity in the square brackets does not have poles for energies $\varepsilon-\mu>\omega_L$, and in the second it does not have poles for energies $\varepsilon-\mu<-\omega_L$. Therefore, with an accuracy of at least $\omega_L/D_{(\lambda)}$ we obtain the following expression for ΔH^p:

$$\Delta H^p(\mathbf{r}\mathbf{r}',\varepsilon)=\sum_L V_L^*(\mathbf{r})g^{\rm opt}(\mathbf{r}\mathbf{r}',\varepsilon-\tilde{\omega}_L)V_L(\mathbf{r}');$$

$$\tilde{\omega}_L\equiv\omega_L\,{\rm Sgn}(\varepsilon-\mu). \quad (60)$$

This together with (50) leads to the fundamental equation of the CCM for the averaged one-particle Green function in lowest order in the quasiparticle–phonon interaction:

$$g(\mathbf{r}\mathbf{r}',\varepsilon)=g^{\rm opt}(\mathbf{r}\mathbf{r}',\varepsilon)+\sum_L\int g^{\rm opt}(\mathbf{r}\mathbf{r}_1,\varepsilon)V_L^*(\mathbf{r}_1)g^{\rm opt}(\mathbf{r}_1\mathbf{r}_2,\varepsilon-\tilde{\omega}_L)V_L(\mathbf{r}_2)g(\mathbf{r}_2\mathbf{r}',\varepsilon)d\mathbf{r}_1d\mathbf{r}_2. \quad (61)$$

Let us discuss Eq. (61), since some of the conclusions following from its analysis will be used in what follows. After eliminating the angular variables in (61) using (52), (55), and (60) we obtain a system of linear integral equations for the radial Green functions:

$$g_{jl}(rr',\varepsilon)=g_{jl}^{\mathrm{opt}}(rr',\varepsilon)+\sum_{Lj'l'}\bar{\varkappa}_L^2(jl,j'l')\times\int g_{jl}^{\mathrm{opt}}(rr_1,\varepsilon)V(r_1)g_{j'l'}^{\mathrm{opt}}(r_1r_2,\varepsilon-\tilde{\omega}_L)g_{jl}(r_2r',\varepsilon)dr_1dr_2, \quad (62)$$

where $\bar{\varkappa}_L^2=\beta_L^2[(2j+1)(2L+1)]^{-1}\langle jl\|Y_L\|j'l'\rangle^2$, and $\langle\|Y_L\|\rangle$ is the reduced matrix element. An explicit solution of the system (62) can easily be found in the approximation of a quasiparticle–phonon "contact" interaction, i.e., in the approximation

$$du/dr\to\tilde{u}_0\delta(r-R), \quad (63)$$

where $\tilde{u}_0$ is the amplitude of the mean field of the shell model. This approximation corresponds to the assumption that the radial wave functions $\chi_\lambda(r)$ (16) vary little near $r=R$ on a scale on the order of the diffusion length of the shell-model potential. A quantitative criterion for the accuracy of the approximation (63) is the closeness to unity of the parameters

$$\eta_{\lambda\lambda'}=(rdu/dr)^2_{\lambda\lambda'}/[R\tilde{u}_0\chi_\lambda(R)\chi_{\lambda'}(R)]^2, \quad (64)$$

where λ and λ' are the quantum numbers of one-particle states of the discrete (and quasidiscrete) spectrum which combine according to (62) and are close in energy. In the approximation (63) the solution of the system (62) can be written as

$$g_{jl}(rr',\varepsilon)=g_{jl}^{\mathrm{opt}}(rr',\varepsilon)+\left[\sum_{Lj'l'}\tilde{\varkappa}_L^2(jl,j'l')g_{jl}^{\mathrm{opt}}(rR,\varepsilon)g_{j'l'}^{\mathrm{opt}}(RR,\varepsilon-\tilde{\omega}_L)g_{jl}^{\mathrm{opt}}(Rr',\varepsilon)\right]\times\left[1-\sum_{Lj'l'}\tilde{\varkappa}_L^2(jl,j'l')g_{jl}^{\mathrm{opt}}(RR,\varepsilon)g_{j'l'}^{\mathrm{opt}}(RR,\varepsilon-\tilde{\omega}_L)\right]^{-1}, \quad (65)$$

where $\tilde{\varkappa}_L^2=(R\tilde{u}_0\bar{\varkappa}_L)^2$. This expression is simplified considerably in the energy interval of interest $|\varepsilon-\varepsilon_\lambda|\ll D_{(\lambda)}$ near the level λ, and also near the levels λ' combining with it according to (65) when the approximate representations (42) and (45) are valid for the Green functions $g_{jl}^{\mathrm{opt}}(rr',\varepsilon)\simeq\chi_\lambda(r)\chi_\lambda(r')g_\lambda^{\mathrm{opt}}(\varepsilon)$. The solution (65) in this approximation can also be written as $g_{jl}(rr',\varepsilon)\simeq\chi_\lambda(r)\chi_\lambda(r')g_\lambda(\varepsilon)$, where

$$g_\lambda(\varepsilon)=g_\lambda^{\mathrm{opt}}(\varepsilon)\left[1-\sum_{L\lambda'}\varkappa_L^2(\lambda,\lambda')g_\lambda^{\mathrm{opt}}(\varepsilon)g_{\lambda'}^{\mathrm{opt}}(\varepsilon-\tilde{\omega}_L)\right]^{-1}. \quad (66)$$

Here $\varkappa_L^2(\lambda,\lambda')=[R\tilde{u}_0\chi_\lambda(R)\chi_\lambda,(R)\bar{\varkappa}_L(jl,j'l')]^2$. It follows from (66) that the strength of the coupling of the quasiparticle λ to the phonon L is determined by the parameter

$$(\nu_{\lambda\lambda'}^L)^2=\varkappa_L^2(\lambda,\lambda')\Big/\frac{1}{2}\Gamma_\lambda^\downarrow\left[(\varepsilon_\lambda-\varepsilon_{\lambda'}-\tilde{\omega}_L)^2+\left(\frac{1}{2}\Gamma_\lambda^\downarrow\right)^2\right]^{1/2}. \quad (67)$$

If $(\nu_{\lambda\lambda'}^L)^2\ll 1$ for all the states λ' combining with the state λ according to the selection rules, then the quasiparticle–phonon coupling is weak, and the width of the one-quasiparticle resonance in the strength function $S_\lambda(\varepsilon)$ is mainly determined by the imaginary part of the optical potential. We note that most of the particle–hole type of configurations are such phonons, and their coupling to the quasiparticles is explicitly taken into account in the approaches of Refs. 3 and 4. In the method discussed here the coupling of quasiparticles to these configurations is taken into account in terms of the imaginary part of the optical potential.

In "soft" spherical nuclei (nuclei with relatively large dynamical quadrupole deformation parameter β_2) at quasiparticle energies which are not too high the parameters $\nu_{\lambda\lambda}^{L=2}\equiv\nu_\lambda=\varkappa_2(\lambda,\lambda)/\frac{1}{2}\Gamma_\lambda^\downarrow(j_\lambda>1/2)$ and $\nu_{\lambda\lambda'}^{L=2}$ turn out to be comparable with unity. In these cases the quasiparticle–2^+-phonon coupling is strong. It is this fact which explains the special role of 2^+ phonons in the fragmentation of one-quasiparticle states in "soft" spherical nuclei. We note that at low excitation energies ($\Gamma_\lambda^\downarrow=0$) the strength of the quasiparticle–2^+-phonon coupling is determined by the quantity $\nu_\lambda^0=\varkappa_2(\lambda,\lambda)/\omega_2(j_\lambda>1/2)$, which is the parameter of the perturbation expansion in the quasiparticle–2^+-phonon interaction, so that $\nu_\lambda=(\omega_2/\frac{1}{2}\Gamma_\lambda^\downarrow)\nu_\lambda^0$. Therefore, at sufficiently high excitation energy (depending on $\Gamma_\lambda^\downarrow$) the strong coupling which occurs for quasiparticles with small excitation energy ($\nu_\lambda^0\gtrsim 1$) can become weak coupling ($\nu_\lambda\ll 1$). In the case of 3^- phonons $\nu_{\lambda\lambda}^{L=3}=0$, and the parameters $\nu_{\lambda\lambda'}^{L=3}$, as a rule, are small, so that the quasiparticle–3^--phonon interaction, as a rule, is weak.

Let us study an example of solving the problem of the one-quasiparticle strength function in the CCM. For this we use the approximation in which the nondiagonal terms in (62) are neglected (the "self-interaction" approximation or the approximation of a single j level,[5] when $L^\pi=2^+$, $\lambda'=\lambda$). According to (8) and (66), we have

$$S_\lambda(\varepsilon) = -\frac{1}{\pi}\,\mathrm{Sgn}(\varepsilon-\mu)\,\mathrm{Im}\,g_\lambda(\varepsilon)$$

$$= -\frac{1}{\pi}\,\mathrm{Im}\left[\varepsilon - \tilde{\varepsilon}_\lambda + \frac{\mathrm{i}}{2}\tilde{\Gamma}^{\downarrow}_\lambda - \frac{\varkappa_2^2(\lambda,\lambda)}{\varepsilon - \tilde{\varepsilon}_\lambda - \tilde{\omega}_2 + \frac{\mathrm{i}}{2}\tilde{\Gamma}^{\downarrow}_\lambda}\right]^{-1} \mathrm{Sgn}(\varepsilon-\mu). \qquad (68)$$

The energy dependence of the strength function $S_\lambda(\varepsilon)$ obviously displays two comparable maxima if $\varkappa_2(\lambda,\lambda) \gtrsim \frac{1}{2}\Gamma^{\downarrow}_\lambda > \omega_2$. The nature of these maxima is easily understood by studying the limiting case $\Delta H^m \to 0$, when $g_\lambda(\varepsilon) \to G_\lambda(\varepsilon)$. According to the spectral expansion (3),

$$G_\lambda(\varepsilon) = \sum_n w_n^\lambda [\varepsilon - \varepsilon_n^\lambda + \mathrm{i}0\,\mathrm{Sgn}(\varepsilon-\mu)]^{-1}, \qquad (69)$$

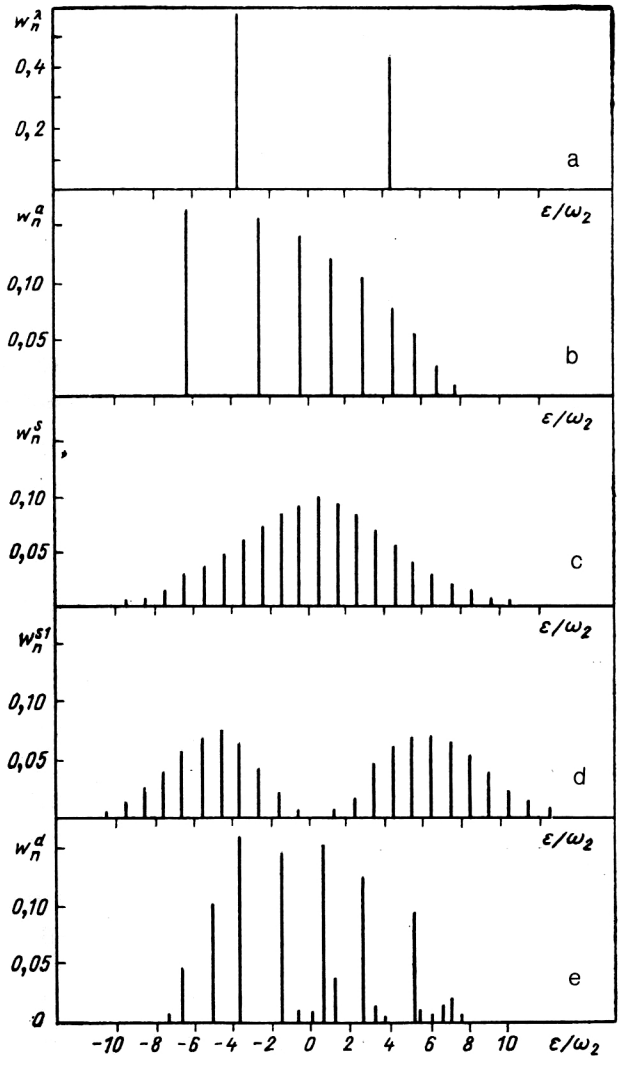


FIG. 1. Distributions of one-quasiparticle (phononless) and one-phonon strengths found using various approximations with $\nu^0 = 4$.

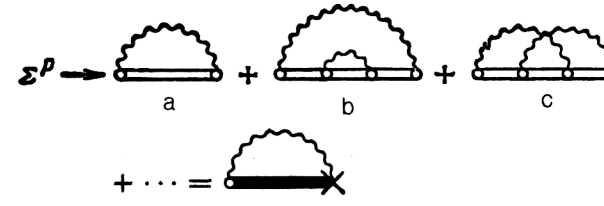
where the quantities w_n^λ have the meaning of the probabilities of finding the phononless state among the exact states of the system. It follows from (68) and (69) that in the limit $\Delta H^m \to 0$ finding the poles and residues of the Green function is equivalent to solving the two-level problem, i.e., the problem of the mixing of the quasiparticle and quasiparticle + 2^+-phonon configurations. In the case $\nu_2^0 \gg 1$, $w_{1,2}^\lambda = 1/2$ and $\varepsilon_{1,2}^\lambda - \varepsilon_\lambda = \pm\varkappa_2(\lambda,\lambda)$. The quantities $w_{1,2}^\lambda$ and $\varepsilon_{1,2}^\lambda - \varepsilon_\lambda$ for the value $\nu_2^0 = 4$ are shown in Fig. 1a.

The two-peak dependence $S_\lambda(\varepsilon)$ (68) is the result of using perturbation theory in V_2 in the equation for the averaged one-particle Green function in the case of strong quasiparticle – 2^+-phonon coupling. However, in the case of strong coupling it is obviously necessary to take into account the contribution to the irreducible self-energy part from configurations containing more phonons. A unique situation arises: when $\nu_2 \ll 1$, the coupling of the channels has practically no effect on the strength function, and for $\nu_2 \gtrsim 1$ it is impossible to restrict ourselves to including only the contribution of one-phonon configurations to $\langle\Sigma^p\rangle$. In other words, the commonly used one-phonon variant of the coupled-channel method has practically no region of applicability. The rest of this review is devoted to the formulation, analysis, and application of an approximate version of the multiphonon variant of the coupled-channel method (MVCCM).

3. THE MULTIPHONON VARIANT OF THE COUPLED-CHANNEL METHOD

The fundamental relations of the MVCCM

As shown above, in the case of strong quasiparticle–phonon coupling it is necessary to include, in addition to the simplest diagram, the contribution to the irreducible self-energy term Σ^p of the more complicated diagrams shown here in the quasiparticle–phonon channel:



Here, as before, the double line corresponds to the Green function G^m (56). By definition, Σ^m in (56) does not include diagrams corresponding to channel coupling, but it does contain diagrams which are irreducible in the quasiparticle–phonon channel. Owing to the small statistical weight of these diagrams, it can be expected that the change of Σ^m due to the phonon contribution will be small. This perturbation series for Σ^p can be formally summed. The heavy line in the resulting diagram corresponds to the exact Green function G, and the cross corresponds to the exact (renormalized) vertex $\tilde{V}_L$. In the case of phonons

with nonzero angular momentum it is not possible to determine the coupling of the vertex to the exact Green function, and so to analyze the effect of renormalization it is necessary to resort to various approximations.[5] The simplest is the assumption that the effect of the vertex renormalization makes a small contribution to Σ^p or ΔH^p. This approximation corresponds to the inclusion of the contribution to Σ^p from only "rainbow" diagrams, i.e., diagrams (a), (b), etc., while neglecting diagrams (c), etc. The accuracy of this approximation is discussed in the following section. Here we formulate the fundamental equations of the MVCCM, an approach in which the coupling of quasiparticles to phonons and multiquasiparticle configurations is taken into account in an energy-averaged way, neglecting the renormalization of the quasiparticle–phonon vertex. In connection with the summation of the diagrams in Σ^p, we note that in the MVCCM the question of classifying multiphonon states on the basis of the angular-momentum addition scheme does not arise. According to the resulting diagram for $\Sigma_L(\mathbf{r}\mathbf{r}',\varepsilon)$, in this approximation the expressions (58) and (59) are valid if the replacement $G^m \to G$ is made in them. Therefore, the quantity $\Delta H^p(\mathbf{r}\mathbf{r}',\varepsilon)$ is determined by an expression of the form (60):

$$\Delta H^p(\mathbf{r}\mathbf{r}',\varepsilon)=\sum_L V_L^*(\mathbf{r})g(\mathbf{r}\mathbf{r}',\varepsilon-\tilde{\omega}_L)V_L(\mathbf{r}'). \qquad (70)$$

When this expression is taken into account, the nonlinear equation for the averaged one-particle Green function $g(\mathbf{r}\mathbf{r}',\varepsilon)$ follows from (50). On the basis of (70) we also obtain the expression for the radial part of ΔH^p:

$$\Delta H^p_{jl}(rr',\varepsilon)=V(r)V(r')\sum_{Lj'l'}\bar{\kappa}^2_L(jl,j'l')g_{j'l'}(rr',\varepsilon-\tilde{\omega}_L), \qquad (71)$$

where the quantities $\bar{\kappa}^2_L(jl,j'l')$ are defined below Eq. (62). The system of nonlinear integral equations for the radial Green functions $g_{jl}(rr',\varepsilon)$ follows from (52) and (71). The solution of this system in the "contact"-interaction approximation (63) can be written down using Eq. (65) with the replacement

$$g^{\rm opt}_{jl}(RR,\varepsilon-\tilde{\omega}_L)\to g_{jl}(RR,\varepsilon-\tilde{\omega}_L), \qquad (72)$$

where the quantities $g_{jl}(RR,\varepsilon)\equiv g_{jl}(\varepsilon)$ are defined by a system of nonlinear functional equations:

$$g_{jl}(\varepsilon)=g^{\rm opt}_{jl}(\varepsilon)+\sum_{Lj'l'}\tilde{\kappa}^2_L(jl,j'l')g^{\rm opt}_{jl}(\varepsilon)g_{j'l'}(\varepsilon-\tilde{\omega})g_{jl}(\varepsilon), \qquad (73)$$

where, as before, $\tilde{\kappa}_L=R\tilde{u}_0\bar{\kappa}_L$. Equations (52) and (71) are the fundamental relations of the MVCCM. In the approximation of the quasiparticle–phonon contact interaction these equations become Eqs. (65), (72), and (73).

Qualitative analysis of the fundamental relations

Let us again turn to the problem of a quasiparticle located in an isolated j level and interacting with 2^+ phonons (the "self-interaction" approximation). In this case $g_{jl}(rr',\varepsilon) \to g_\lambda(\varepsilon)\chi_\lambda(r)\chi_\lambda(r')$, and the equation for $g_\lambda(\varepsilon)$ in the limit $\Delta H^m \to 0$ [when $g_\lambda(\varepsilon) \to G^a_\lambda(\varepsilon)$] has, according to (65) and (72), a form analogous to (73) (for definiteness we consider the particle Green function when $\tilde{\omega}_2 \to \omega_2$):

$$G^a_\lambda(\varepsilon)=G_{0\lambda}(\varepsilon)+\kappa_2^2(\lambda,\lambda)G_{0\lambda}(\varepsilon)G^a_\lambda(\varepsilon-\omega_2)G^a_\lambda(\varepsilon), \qquad (74)$$

where the constant $\kappa_2(\lambda,\lambda)\equiv\kappa_2$ is defined after Eq. (66). The solution of the functional equation (74) can be found in Ref. 5 (here and below in this section the particle energy is taken to be zero):

$$G^a_\lambda(\varepsilon)=\frac{1}{\kappa_2}\frac{J_{-q}(x)}{J_{-q-1}(x)};\quad q=\varepsilon/\omega_2;\quad x=-2\kappa_2/\omega_2, \qquad (75)$$

where $J_q(x)$ is the Bessel function. According to (69), the poles and residues of the Green function (75), respectively, determine the energy levels ε^a_n and the one-quasiparticle strength distribution w^a_n in the approximation in which the renormalization of the quasiparticle–phonon vertex is neglected. As an example, in Fig. 1b we show the distribution w^a_n calculated using (75) for the value $\nu^0_2=4$.

In Eq. (74) we drop the indices λ and L characterizing the quasiparticle quantum numbers and the phonon angular momentum ($\kappa_2 \to \kappa, \omega_2 \to \omega, \nu^0_2 \to \nu^0, G_\lambda \to G$). Neglecting the vertex renormalization, this equation formally corresponds to the problem of a particle emitting and absorbing scalar (0^+) phonons without changing its quantum state. This problem can also be solved exactly (i.e., taking into account the vertex renormalization) by two methods. The first consists of solving the equation for the one-particle Green function $G^s(\varepsilon)$ in which the vertex renormalization is taken into account using the Ward difference identity (see, for example, Ref. 5):

$$\tilde{V}/V=[(G^s(\varepsilon)^{-1}-(G^s(\varepsilon-\omega))^{-1}]/\omega. \qquad (76)$$

The equation for the Green function $G^s(\varepsilon)$ [which can be obtained from (74) by the replacement $\kappa^2 \to \kappa^2\tilde{V}/V$] is linearized using (76):

$$G^s(\varepsilon)=G_0(\varepsilon)+\frac{\kappa^2}{\omega}G_0(\varepsilon)\{G^s(\varepsilon-\omega)-G^s(\varepsilon)\}. \qquad (77)$$

The solution of (77) can easily be found: the energy levels of the "particle + scalar phonons" system are equally spaced, and the one-particle strength is distributed according to a Poisson distribution:[5]

$$\varepsilon^s_n=(n-(\nu^0)^2)\omega;\quad w^s_n=e^{-(\nu^0)^2}(\nu^0)^{2n}/n! \qquad (78)$$

The other solution method consists of diagonalization, so that the Hamiltonian of the problem becomes

$$\mathcal{H}=\omega a^+a+\kappa b^+b(a+a^+). \qquad (79)$$

Here a^+ (a) and b^+ (b) are, respectively, the phonon and particle creation (annihilation) operators with the condition $b^+b \to 1$. According to (79), the problem of finding the energies ε^s_n and the probabilities w^s_n is formally equivalent to that of the excitation of a harmonic oscillator in the ground state when a uniform external field is suddenly switched on. The solution of this problem is well known[21] and also leads to Eqs. (78). The distribution w^s_n for the

TABLE I. Dimensionless moments of the distributions w_n^s, w_n^a, and w_n^d for $\nu^0=4$.

Distribution	σ_0	σ_1	σ_2	σ_3	σ_4
w_n^s	1	0	16	16	784
w_n^a	1	0	16	16	528
w_n^d	0.97	−0.20	15	10	490

value $\nu^0 = 4$ is shown in Fig. 1c. By a similar method we can find the distribution of the weight of the one-phonon component of the exact states of the Hamiltonian (79), w_n^{s1}:

$$w_n^{s1}=\frac{e^{-(\nu^0)^2}}{(\nu^0)^2}\frac{(\nu^0)^{2n}}{n!}(n-(\nu^0)^2)^2. \tag{80}$$

The distribution of the quantities w_n^{s1} has two comparable maxima if $\nu^0 \gg 1$ (see Fig. 1d; $\nu^0 = 4$).

The two distributions studied, w_n^s and w_n^a, correspond to limiting cases of the problem of an isolated j level. For example, for quasiparticle angular momentum $j \gg 2$, the 2^+ phonons can be assumed to be scalar, and the one-quasiparticle strength distribution can be analyzed using the Green function $G^s(\varepsilon)$ and the distribution w_n^s (78). In the case of small j ($j \leqslant 2$) it can be expected that the vertex is weakly renormalized owing to the kinematic suppression of the contribution of the corresponding graphs of perturbation theory,[5] so that the distribution w_n^a can be used. In relation to this the results of calculations of the strength distribution of the $E1$ giant resonance (the $E1$ GR) owing to coupling to 2^+ phonons are interesting. The result of solving the corresponding diagonalization problem on a basis of states including up to 14 2^+ phonons is given on p. 407 (Russian ed.) of Ref. 17. The $E1$-GR strength distribution w_n^d for $\nu^0 = 4$ is given in Fig. 1e.

As follows from Figs. 1b and 1c, the distributions w_n^a and w_n^s differ considerably from each other. For example, the energy intervals between neighboring levels with energies near ε_λ differ by about a factor of two, the distribution w_n^a is asymmetric, and so on. It should, however, be borne in mind that for sufficiently large quasiparticle energies $[(1/2)\Gamma^\downarrow > \omega]$ only the averaged distributions $\bar{w}_n$ are observed. Information on these distributions is contained in the moments σ_k:

$$\left.\begin{aligned}\sigma_0&=\sum_n w_n;\ \sigma_1=\sum_n \varepsilon_n w_n/\omega\equiv\bar{\varepsilon}/\omega;\\ \sigma_{k\geqslant 2}&=\sum_n (\varepsilon_n-\bar{\varepsilon})^k w_n/\omega^k.\end{aligned}\right\} \tag{81}$$

For example, σ_0 determines the normalization of the distribution, σ_1 determines the average energy, σ_2 determines the rms dispersion (the fragmentation width $\Gamma_{ph}^\downarrow \equiv 2.35\sigma_2^{1/2}\omega$), σ_3 is the asymmetry of the distribution, and so on. The moments of the distributions w_n^s and w_n^a as functions of the parameter ν^0 are easily calculated exactly. In Table I we give the results of calculations of the moments $\sigma_{k\leqslant 4}$ of the distributions w_n^a, w_n^s, and w_n^d for the value $\nu^0 = 4$. We note that it follows from (75) and (78) (see also Table I) that the effective number of phonons n_{eff} which should be included in studying the one-quasiparticle strength distribution is determined by the dimensionless coupling constant ν_0:

$$n_{\text{eff}}\equiv\Gamma_{ph}^\downarrow/\omega\sim\sigma_2^{1/2}=\nu^0. \tag{82}$$

It follows from the data of Table I that the averaged distributions $\bar{w}_n^s$ and $\bar{w}_n^a$ (and to a large extent $\bar{w}_n^a$ and $\bar{w}_n^d$) can be expected to be close. To verify this directly, we turn to the energy-averaged Green functions $g^a(\varepsilon) = G^a(\varepsilon + iI')$, $g^s(\varepsilon) = G^s(\varepsilon + iI')$ $(I' > \omega)$. In the limit $\Gamma \gg \omega$, where Γ is the total width of the strength function (it is precisely the case, which corresponds to strong quasiparticle–phonon coupling, that is interesting from the viewpoint of determining whether or not it is possible to neglect the vertex renormalization), the equations for the averaged Green functions follow from (74) and (77) with accuracy ω/Γ:

$$g^a(\varepsilon)=g_0(\varepsilon)+\varkappa^2 g_0(\varepsilon)[g^a(\varepsilon)]^2; \tag{83}$$

$$g^s(\varepsilon)=g_0(\varepsilon)-\varkappa^2 g_0(\varepsilon)dg^s(\varepsilon)/d\varepsilon, \tag{84}$$

where $g_0(\varepsilon) = (\varepsilon + iI')^{-1}$. The solutions of these equations can be found analytically:

$$g_0^a(\varepsilon)=(\varepsilon+iI')[1-\sqrt{1-4\varkappa^2/(\varepsilon+iI')^2}]/2\varkappa^2; \tag{85}$$

$$g^s(\varepsilon)=-\left\{\frac{i}{\varkappa}\sqrt{\frac{\pi}{2}}(1-\operatorname{erf}(I'/\sqrt{2}\varkappa))+\frac{1}{\varkappa^2}\int_0^\varepsilon e^{(x+iI')^2/2\varkappa^2}dx\right\}e^{-(\varepsilon+iI')^2/2\varkappa^2}, \tag{86}$$

where $\operatorname{erf}(z)$ is the error function. It follows from these relations that the strength functions $\bar{w}^{a,s}(\varepsilon) = -(1/\pi)\operatorname{Im} g^{a,s}(\varepsilon)$ are close. As an illustration of this statement, in Figs. 2 and 3 we show the strength functions for the parameters $\nu' = 1$ and $\nu' = 5$ $(\nu' = \varkappa/I')$. Therefore, the one-particle strength distribution averaged over the energy range $I' > \omega$ in the problem of the interaction with scalar phonons depends weakly on the inclusion of the vertex renormalization.

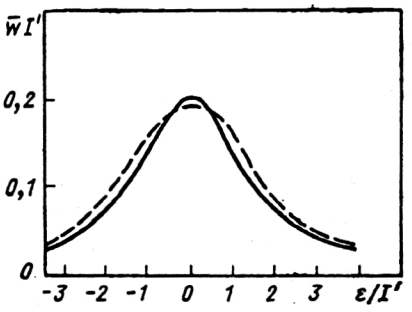


FIG. 2. Calculated strength functions $\bar{w}^s(\varepsilon)$ (solid line) and $\bar{w}^a(\varepsilon)$ (dashed line) for $\nu' = 1$.

Turning now to discussion of the damping of one-quasiparticle states with large excitation energy, we note two features favoring the use of Eq. (73) for the Green func-

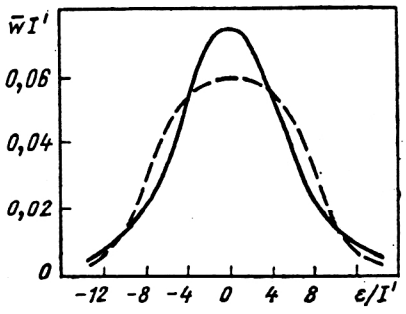


FIG. 3. The same as in Fig. 2 for $\nu' = 5$.

tion $g_{ji}(rr',\varepsilon)$ with an unrenormalized quasiparticle–phonon vertex: (1) the use of the optical model to describe the coupling of the one-quasiparticle states with multiparticle ones actually implies analysis of the one-quasiparticle strength distribution averaged over the energy interval I ($w_{\lambda\lambda} > I$); (2) in the practically important case of 2^+ phonons, where the role of neighboring levels with the same parity in the relaxation of the one-quasiparticle state is noticeable, the vertex-renormalization effect is diminished, both owing to the additional kinematic suppression of the corresponding graphs due to the random phases of the vector-addition coefficients,[5] and owing to the additional fragmentation of the one-quasiparticle strength and, consequently, the decrease of the effective averaging interval.

The arguments presented in this section permit us to use the fundamental relations of the MVCCM, (50) and (70), or (52) and (71), in which the renormalization of the quasiparticle–phonon vertex is neglected, for the quantitative analysis of the observable consequences of the relaxation of the one-particle degree of freedom for sufficiently high excitation energy.

The variant of the MVCCM described here assumes that the phonons correspond to harmonic oscillations. It can be expected that after averaging over the energy interval $I' > \omega$ the effect of anharmonicity will be insignificant for $(\Delta\omega)_{\text{eff}} < \varkappa \sim \Gamma^{\downarrow}_{ph}$. Here $(\Delta\omega)_{\text{eff}}$ is the energy shift of real n_{eff} phonon states relative to the energy of the state containing n_{eff} harmonic phonons. If we use the estimate $(\Delta\omega)_{\text{eff}} = (1/2)n^2_{\text{eff}}\Delta\omega$ ($\Delta\omega$ is the energy shift of the two-phonon multiplet),[17] this condition becomes the inequality $\Delta\omega/\omega < \omega/\varkappa$, which as a rule is satisfied for vibrational nuclei.

To conclude this section, we note that, since phonons are not true bosons, in describing multiphonon configurations the question of taking into account the Pauli principle can arise. In connection with this we introduce the parameter n_c, which has the meaning of the number of particle–hole configurations forming the phonon and thereby characterizing the degree of collectivization of the excited state corresponding to the phonon: $n_c = B(E2)/B_{sp}(E2)$, where $B(E2)$ is the reduced phonon excitation probability and $B_{sp}(E2)$ is the corresponding one-particle estimate. In "soft" nuclei $n_c \sim$ 20–40 (Ref. 22). As a rule, $n_{\text{eff}} \ll n_c$, so that with accuracy n_{eff}/n_c the effect of the Pauli principle in the description of multiphonon states in the MVCCM can be neglected.

Choice of the model parameters. Method of solving the system of equations for the radial Green functions

According to the fundamental relations of the MVCCM, the calculation of the averaged one-particle Green function requires the specification of the same parameters as in the usual versions of the CCM: (1) the parameters of the shell-model potential (15); (2) the parameters of the optical addition to the shell-model potential (34); and (3) the parameters corresponding to the dynamical deformation β_L and the phonon energy ω_L.

In the calculations whose results are given below, the parametrization of the shell-model potential in (15) is chosen in the form

$$u(r) = u_0(r) + \tfrac{1}{2}\tau^{(3)}v(r) + \tfrac{1}{2}(1-\tau^{(3)})u_c(r), \quad (87)$$

where $u_0(r)$ is the isoscalar part of the potential, $v(r)$ is the symmetry energy proportional to the neutron excess, $u_C(r)$ is the energy of the Coulomb interaction of the proton with the nucleus, and $\tau^{(3)}$ is the isotopic matrix. In addition,

$$u_0(r) = -V_0 f(r,R,a);$$
$$u_{so}(r) = V_{so}\frac{\Lambda^2}{r}\frac{df}{dr};\ \frac{1}{2}v(r) = \alpha u_0(r)\frac{N-Z}{A}; \quad (88)$$
$$f(r,R,a) = \left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1};$$
$$u_C(r) = \begin{cases} \dfrac{Ze^2}{2R_C}\left[3-\left(\dfrac{r}{R_C}\right)^2\right], & r \geqslant R_C, \\ \dfrac{Ze^2}{r}, & r > R_C, \end{cases} \quad (89)$$

where $R = r_0 A^{1/3}$ and $R_C = r_{0C}A^{1/3}$. We note that in (63) $\tilde{u_0} = V_0[1 \pm \alpha(N-Z)/A]$. The parameters of the shell-model potential, the use of which makes it possible to reproduce the experimental nucleon binding energy B^{exp} in the average over A, were chosen as in Ref. 23:

$$\left.\begin{aligned} &V_0 = 53.3 \text{ MeV};\ r_0 = r_{0C} = 1.24 \text{ F};\ a = 0.63 \text{ F}; \\ &V_{so} = 14.02[1+2(N-Z)/A] \text{ MeV};\ \Lambda = 1.41 \text{ F}; \\ &\alpha = -0.63. \end{aligned}\right\} \quad (90)$$

The parametrization of ΔH^m (34) was chosen in the form usually used in calculations based on the optical model or the CCM:

$$w(r,|\varepsilon-\mu|) = -4w(|\varepsilon-\mu|)adf/dr,$$
$$\Delta(r,\varepsilon-\mu) = \Delta(\varepsilon-\mu)f, \quad (91)$$

where $w(|\varepsilon-\mu|)$ and $\Delta(\varepsilon-\mu)$ are variable functions. In the calculations of the one-quasiparticle and neutron strength functions the quantities $w(|\varepsilon-\mu|) = \text{const}$ were chosen to be 1.0–1.5 MeV and $\Delta = 0$ (there are no independent data on the value of the parameter Δ for energies

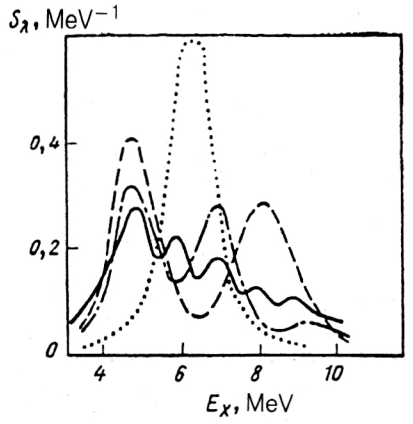


FIG. 4. Results of calculations of the strength function of the $(1g_{9/2})_n^{-1}$ state in the ^{123}Te nucleus in the self-interaction approximation: dotted line—calculation using the optical model; dashed line—one-phonon approximation; dot–dash line—two-phonon approximation; solid line—results of the exact (in the number of phonons) approximation.

$E_x \sim B$). This choice of imaginary part is due to the fact that by using values $w \simeq 1$–2 MeV it is possible to satisfactorily describe the experimental data on the neutron strength functions and the cross sections for low-energy neutron scattering by nuclei in a wide range of atomic numbers using simple versions of the CCM (see, for example, Ref. 26). In connection with this we note that in the case of strong quasiparticle–phonon coupling the width of the strength function depends weakly on w when $\Gamma_{ph} > w$ (see Fig. 5c).

The energy to excite an odd nucleus from the ground-state configuration λ_0^1 (or λ_0^{-1}) is determined by taking into account nucleon pairing according to the expression ($E_x > \tilde{\Delta}$):

$$E_x = |\varepsilon - \mu| - [(\varepsilon_{\lambda_0} - \mu)^2 + \tilde{\Delta}^2]^{1/2}, \qquad (92)$$

where the parameter $\tilde{\Delta}$ can be found from the difference between the experimental values of the nucleon binding energy in the given odd nucleus and the neighboring even nuclei,[24] and the chemical potential μ is found from the standard equations[19] taking into account all the discrete levels in the shell-model potential. The parameters describing the dynamical deformation β_L and the phonon energy ω_L were taken from Ref. 25.

To find the radial Green functions $g_{jl}(rr',\varepsilon)$ in the approximation of the quasiparticle–phonon contact interaction according to (65) and (72) it is necessary to solve the system of nonlinear functional equations (73) for the functions $g_{jl}(RR,\varepsilon) \equiv g_{jl}(\varepsilon)$. The idea of the method of solution is the following (for definiteness we consider one-hole excitations, when $\tilde{\omega}_L = -\omega_L$, with the value of L fixed). For energies $\varepsilon \simeq \varepsilon_{0jl}$ near the antibound state [$(\varepsilon_{0jl} - \varepsilon_\lambda) \sim Dg/2$] the coupling of the one-quasiparticle state to phonons is suppressed by the ratio $(2\varkappa_L/D_{(\lambda)})^2 \ll 1$, as follows from the system (73). Therefore to this accuracy it can be assumed that $g_{jl}(\varepsilon_{0jl}) \simeq g_{jl}^{\rm opt}(\varepsilon_{0jl})$. Then from (73)

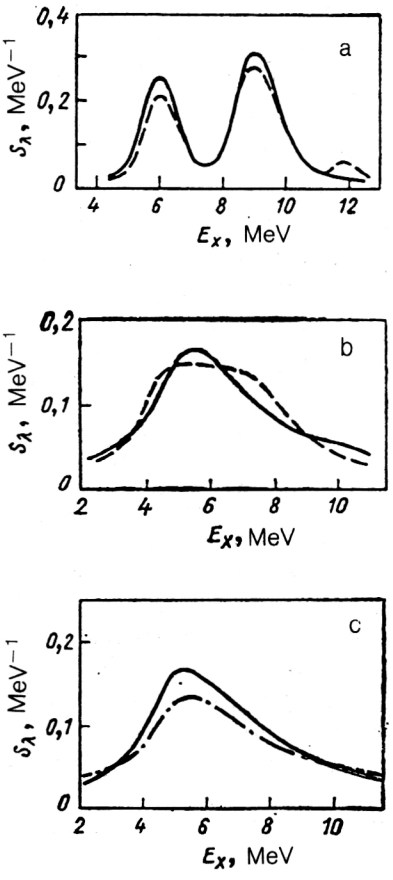


FIG. 5. Results of calculations of the strength function of the $(1g_{9/2})_n^{-1}$ state in the ^{123}Te nucleus: (a) in the one-phonon approximation taking into account quasiparticle coupling to only 2^+ (solid line) and to 2^+ and 3^- (dashed line) phonons; (b) in the one-phonon approximation in the case of "self-interaction" using the contact and realistic form factors of the quasiparticle–2^+-phonon interaction (solid and dashed lines, respectively); (c) using the values $w = 1$ MeV (solid line) and $w = 2$ MeV (dot–dash line).

we can find $g_{jl}(\varepsilon_{0jl} - \omega_L)$, $g_{jl}(\varepsilon_{0jl} - 2\omega_L)$, and so on. In practice, the chain of equations

$$g_{jl}(\varepsilon) = g_{jl}^{\rm opt}(\varepsilon) \times \left[1 - \sum_{j_1 l_1} \tilde{\varkappa}_L^2(jl, j_1 l_1) g_{jl}^{\rm opt}(\varepsilon) g_{j_1 l_1}(\varepsilon + \omega_L)\right]^{-1};$$

$$g_{j_1 l_1}(\varepsilon + \omega_L) = g_{j_1 l_1}^{\rm opt}(\varepsilon + \omega_L)\left[1 - \sum_{j_2 l_2} \tilde{\varkappa}_L^2(j_1 l_1, j_2 l_2) \times g_{j_1 l_1}^{\rm opt}(\varepsilon + \omega_L) g_{j_2 l_2}(\varepsilon + 2\omega_L)\right]^{-1}; \qquad (93)$$

$$g_{j_m l_m}(\varepsilon + m\omega_L) = g_{j_m l_m}^{\rm opt}(\varepsilon + m\omega_L) \times \left[1 - \sum_{j_i l_i} \tilde{\varkappa}_L^2(j_m l_m, j_i l_i) g_{j_m l_m}^{\rm opt}(\varepsilon + m\omega_L) g_{j_i l_i}(\varepsilon + (m+1) + \omega_L)\right]^{-1}$$

is terminated by the substitution $g_{j_1l_1} \to g^{\text{opt}}_{j_1l_1}$ near the energy $\varepsilon' = \varepsilon + m(\varepsilon)\omega_L \simeq \varepsilon_{0jl}$ at which the

$$Q_{Lj'l'}(\varepsilon') = \left| \sum_{j_1l_1} \tilde{\varkappa}^2_L(j'l', j_1l_1) g^{\text{opt}}_{j'l'}(\varepsilon') g^{\text{opt}}_{j_1l_1}(\varepsilon' + \omega_L) \right| \tag{94}$$

do not exceed by a small amount the given value of ξ for all the values of ε of interest near the one-quasiparticle resonance λ in question and all the one-particle states λ' combining with each other and with the state λ according to the selection rules. In the case of 2^+ phonons in actual calculations of the strength functions of states with normal parity (the parity of the shell) all the states within this shell are taken into account, and in calculations of the strength functions of states of anomalous parity all the states of the "parent" shell are included. In the case of 3^- phonons the situation is reversed. In the calculations the value of ξ was taken to be 0.01. We note that the replacement $g(\varepsilon + n\omega_L) \to g^{\text{opt}}(\varepsilon + n\omega_L)$ in (93) for any value of ε corresponds to the inclusion of the contribution of k-phonon configurations ($k \leqslant n$) in the analysis of the fragmentation of the one-quasiparticle strength. The effective number of phonons is taken to be the value $n = n_{\text{eff}}$ beginning at which the solution of the system of algebraic equations (93) for $g_{jl}(\varepsilon)$ is practically independent of n for the values of ε of interest. Naturally, this value is consistent with the estimate $n_{\text{eff}} \simeq \nu^0$.

If we do not use the approximation of the quasiparticle–phonon contact interaction (63) but use the realistic form factor of the interaction $V(r)$ (55), to find the radial Green functions $g_{jl}(rr', \varepsilon)$ it is necessary to solve the system of integral equations (52) and (71). The solution of this system can be found using a procedure of the form (93) with the essential difference that at each energy "step" it is necessary to solve a system of linear integral equations rather than algebraic equations.

The equations formulated in this section for the averaged one-particle Green function, the method of solving the system of nonlinear equations for the radial Green functions, and also our qualitative analysis of the approximations that we have used make it possible to proceed to a quantitative MVCCM interpretation of the observable consequences of relaxing the one-quasiparticle degree of freedom in soft spherical nuclei.

4. MVCCM ANALYSIS OF ONE-QUASIPARTICLE STRENGTH FUNCTIONS IN SPHERICAL NUCLEI OF INTERMEDIATE WEIGHT

Numerical analysis of the various approximations

The quantitative analysis of the one-quasiparticle strength functions $S_{jl}(E_x)$ using the MVCCM is based on (8). The choice of model parameters needed for calculating the averaged one-particle Green function is discussed in Sec. 3. The results of calculations of the one-quasiparticle strength functions given below were obtained, unless stated otherwise, using the approximation of the quasiparticle–phonon contact interaction (63) with energy-independent strength of the imaginary part of the optical potential (91), $w = 1.0$ MeV. As an example illustrating the various approximations, in Figs. 4 and 5 we show the results of calculations of the absolute value and the energy dependence of the strength function $S_{jl}(E_x)$ for the neutron-hole configuration $(1g_{9/2})^{-1}_n$ in the ^{123}Te nucleus, the coupling of which to the 2^+ phonon is strong ($\nu^0_2 = 2.73$). In Fig. 4 we show the results of calculations carried out in the self-interaction approximation including configurations containing up to n 2^+ phonons: $0 \leqslant n \leqslant 2$, and also the results of the "exact" (in the number of phonons) calculation. The case $n = 0$ corresponds to the optical model, $n = 1$ corresponds to the one-phonon approximation or the CCM in the self-interaction approximation, and $n = 2$ corresponds to the solution of the system (73) in the two-phonon approximation. The exact calculation corresponds to the exact (in the number of phonons) solution of the system (73) in the self-interaction approximation. In Fig. 5 we show the results of the calculations of the one-quasiparticle strength function for the same configuration in the approximation in which one-particle levels combining with the $1g_{9/2}$ level are taken into account. In the case of 2^+ phonons these are the $3s$, $2d$, and $1g$ levels,

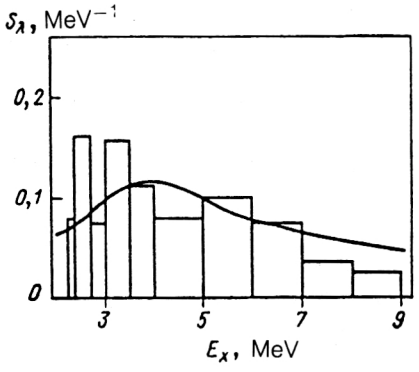


FIG. 6. Results of calculations of the strength function of the $(1g_{9/2})^{-1}_n$ state in the nucleus ^{109}Pd compared with the experimental data (histogram) of Ref. 27.

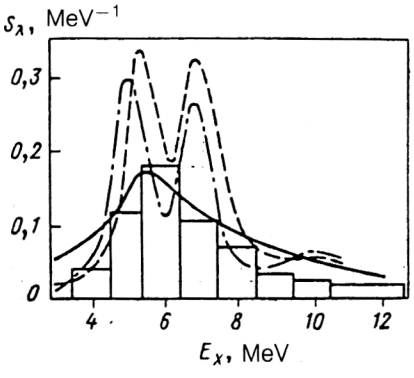


FIG. 7. Results of calculations of the strength function of the $(1g_{9/2})^{-1}_n$ state in the nucleus ^{123}Te (solid line) compared with the experimental data (histogram) of Ref. 33 and with the QPM results (dashed line) of Ref. 34. The dot–dash line shows the results of the calculation using the MVCCM in the two-phonon approximation.

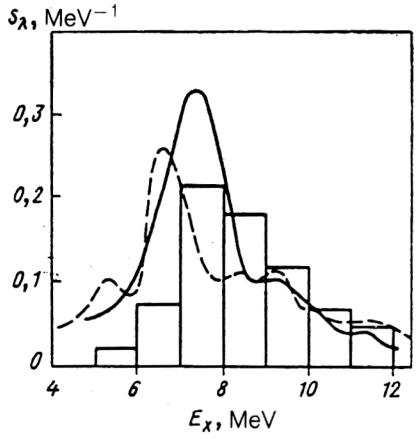


FIG. 8. Results of calculations of the strength function of the $(1g_{9/2})_n^{-1}$ state in the nucleus ^{143}Sm (solid line) compared with the experimental data (histogram) of Ref. 32 and with the QPM results (dashed line) of Ref. 32.

and in the case of 3^- phonons they are the $2p$ and $1f$ levels. These and similar examples in combination with the results of qualitative analysis of the fundamental relations of the CCM and the MVCCM (see Secs. 2 and 3) lead to the following conclusions:

(1) The inclusion of the interaction of the one-quasiparticle state with multiphonon configurations can lead to results which differ qualitatively from both those of optical-model calculations and those obtained in the one-phonon approximation.

(2) As a rule, the self-interaction approximation is not quantitatively accurate: coupling to other states of the same parity leads to noticeable additional fragmentation of the one-quasiparticle state.

(3) The approximation of the quasiparticle–phonon contact interaction ensures sufficient accuracy of the calculations of the strength functions for states with orbital angular momentum $l > 1$.

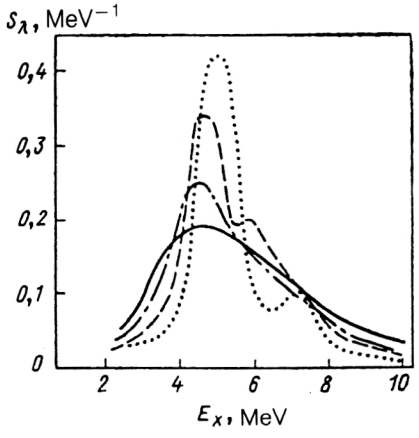


FIG. 9. Results of calculations of the strength function of the $(1g_{9/2})_p^{-1}$ state in isotopes of Pr: dotted line—^{141}Pr; dashed line—^{143}Pr; dot–dash line—^{145}Pr; solid line—^{147}Pr.

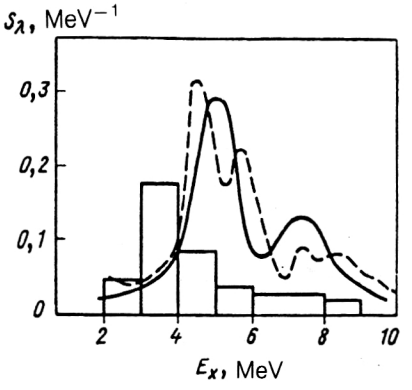


FIG. 10. The same as in Fig. 8 for the $(2f_{7/2})_p$ state in the nucleus ^{145}Eu. The experimental data and QPM results are from Ref. 32.

(4) The interaction of quasiparticles with the 3^- phonon gives, as a rule, a small contribution to the formation of the one-quasiparticle strength functions.

Quantitative analysis of the strength functions. Comparison with the experimental data

For the quantitative analysis of the one-quasiparticle strength functions using the MVCCM we chose one-quasiparticle configurations with energies $E_x \simeq 5$–10 MeV, for which there are experimental data on the cross sections for one-nucleon transfer reactions giving some idea of the one-quasiparticle strength distribution,[27–33] and a number of states for which there are as yet no experimental data. For nuclei except ^{89}Zr and neighboring nuclei of ^{208}Pb the strength functions were calculated with the explicit inclusion of the quasiparticle coupling to only 2^+ phonons, while for these nuclei only the coupling to 3^- phonons was included. The results of the calculations of the absolute value and the energy dependence of the strength functions of several one-quasiparticle states in spherical nuclei are compared with the available experimental data in Figs. 6–13 (see also Refs. 6–8). In Figs. 14 and 15 we show the calculated energy dependences of the "weighted" strength

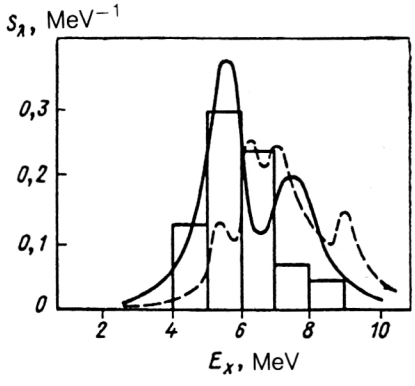


FIG. 11. The same as in Fig. 10 for the $(1h_{9/2})_p$ state.

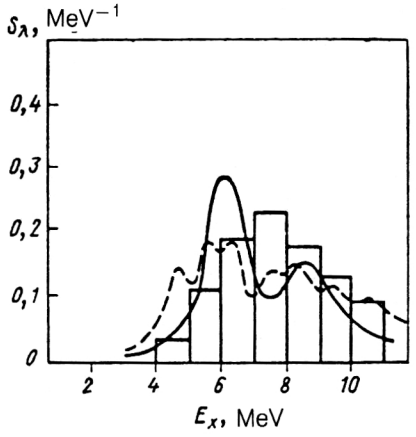


FIG. 12. The same as in Fig. 10 for the $(1i_{13/2})_p$ state.

function of a number of states: a quantity proportional to the cross section of the corresponding one-nucleon transfer reaction:

$$S_w(E_x)=\sum_\lambda (2j+1)S_\lambda(E_x)\Big/\sum_\lambda (2j+1). \qquad (95)$$

In Fig. 15 we compare the calculated values of $S_w(E_x)$ for the $2f_{7/2}$, $1h_{9/2}$, and $1i_{13/2}$ subbarrier one-proton states in the ^{145}Eu nucleus (see also Figs. 10–12) with the corresponding experimental value. The latter was determined according to (95) from the experimental energy dependences of the strength functions of these states.[32] In some of the figures we also give the results of calculations of the strength functions carried out using the quasiparticle–phonon model and the MVCCM in the two-phonon approximation.

For quantitative comparison with the experimental data on the calculated energy dependence of the strength function (in a few cases that of the weighted strength function of several states) we determined the parameters characterizing the one-quasiparticle strength distribution. These parameters are:

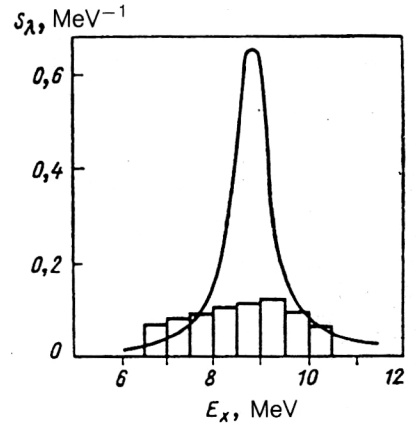


FIG. 13. The same as in Fig. 6 for the $(1h_{11/2})_n^{-1}$ state in the nucleus ^{207}Pb. The experimental data are taken from Ref. 32.

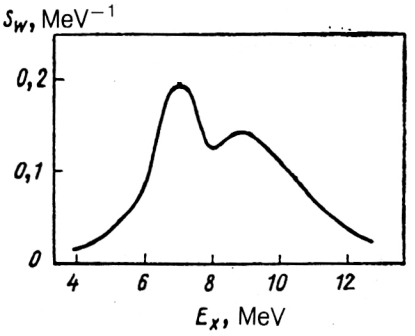


FIG. 14. Results of calculations of the weighted strength function of the $(1h_{9/2})_n$ and $(1i_{13/2})_n$ states in the nucleus ^{113}Sn.

(1) The spectroscopic factors found for the energy range E_1–E_2:

$$S_\lambda=\int_{E_1}^{E_2} S_\lambda(E_x)dE_x, \qquad (96)$$

the average energies

$$\bar{E}_x=\int_{E_1}^{E_2} S_\lambda(E_x)E_x dE_x/S_\lambda, \qquad (97)$$

and the width

$$\Gamma=2.35\left(\int_{E_1}^{E_2} S_\lambda(E_x)(E_x-\bar{E}_x)^2 dE_x/S_\lambda\right)^{1/2}. \qquad (98)$$

(2) The energies of the maxima E_m and the widths Γ_m of one or two Gaussians, which were used to approximate the energy dependence of the strength function $S_\lambda(E_x)$ or $S_w(E_x)$. The quantities thus calculated are given in Tables II–IV, where they are compared with the corresponding experimental data (more detailed information about the calculated energies, widths, and spectroscopic factors and their comparison with the corresponding experimental data can be found in Ref. 8).

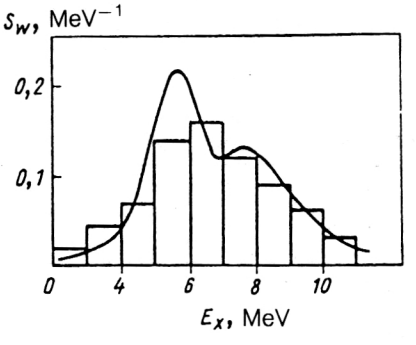


FIG. 15. Results of calculations of the weighted strength function of the $(2f_{7/2})_p$, $(1h_{9/2})_p$, and $(1i_{13/2})_p$ states in the nucleus ^{145}Eu compared with the corresponding experimental data (histogram) of Ref. 32.

TABLE II. Results of calculations of the parameters of the energy dependences of the one-quasiparticle strength functions. Only the quasiparticle interaction with 2^+ phonons is taken into account explicitly.

Nucleus	State	Energy range, MeV	E_x, MeV		Γ, MeV		S_λ	
			Theory	Experiment	Theory	Experiment	Theory	Experiment
^{109}Pd	$(1g_{9/2})_n^{-1}$	2–9	5.46	-	4.73	-	0.55	0.55
	$(2p_{1/2})_n^{-1} + (2p_{3/2})_n^{-1}$	2–9	6.79	-	3.67	-	0.50	0.45
^{111}Sn	$(1g_{9/2})_n^{-1}$	3.4–5.3	4.45	4.7	1.10	1.28	0.46	0.62
^{115}Sn	$(1g_{9/2})_n^{-1}$	3.6–10.5	6.13	6.47	3.47	3.76	0.78	0.89
	$(2p_{1/2})_n^{-1} + (2p_{3/2})_n^{-1}$	2.6–10.5	7.90	6.58	3.23	4.63	0.80	0.45
^{119}Sn	$(1g_{9/2})_n^{-1}$	3.8–6.5	5.51	5.39	1.62	1.55	0.48	0.29
^{121}Te	$(1g_{9/2})_n^{-1}$	3.8–6.0	5.03	5.0	1.41	1.9	0.30	0.46
^{123}Te	$(1g_{9/2})_n^{-1}$	4.2–9.5	6.52	5.5	3.24	3.5	0.60	0.55
^{129}Te	$(1g_{9/2})_n^{-1}$	4–10	7.13	7.5	3.32	4.9	0.74	0.52
^{143}Sm	$(1g_{9/2})_n^{-1}$	6–12	8.01	7.6	3.10	3.0	0.74	0.52
^{145}Eu	$(2f_{7/2})_p$	6–9	5.66	4.3	3.71	1.8	0.70	0.43
	$(1h_{9/2})_p$	3–12	6.44	5.9	3.90	1.23	0.89	0.75
	$(1i_{13/2})_p$	3–12	7.38	7.6	4.41	4.0	0.84	0.54

In Table V we give the quasiparticle–phonon coupling constants and the parameters η (64): in the case of coupling to 2^+ phonons the values of $\nu_2^0 = \varkappa_2(\lambda,\lambda)/\omega_2$ and $\eta_{\lambda\lambda}$, and in the case of coupling to 3^- phonons the maximum of the values $\nu_3^0 = \varkappa_3(\lambda,\lambda')/\omega_3$ determined for all the states λ' combining with the state in question λ in accordance with the selection rules, and also the corresponding parameter $\eta_{\lambda\lambda'}$.

Let us make a few comments about these results.

1. In the MVCCM with explicit inclusion of the quasiparticle interaction with only 2^+ phonons it is possible to satisfactorily describe most of the analyzed experimental data on the strength functions of one-quasiparticle states with sufficiently high excitation energies.

2. In the ^{145}Eu nucleus the experimental and theoretical values of the parameters of the weighted strength function of the $2f_{7/2}$, $1i_{13/2}$, and $1h_{9/2}$ subbarrier one-proton states of similar energy agree better than the parameters of the strength functions of each of these states (Figs. 11-12 and 15). This might indicate that there are errors in the experimental resolution of the contributions to the cross section for the one-nucleon transfer reaction of one-quasiparticle states with similar large angular momenta.

3. The results of the calculations of the strength functions of one-quasiparticle states in near-magic nuclei differ noticeably from the corresponding experimental data. The possible reasons for this discrepancy such as the choice of intensity of the imaginary part of the optical potential and the inclusion of the effect of low-lying 2^+ levels in neighboring nuclei require separate analysis.

4. The energy dependence of the weighted strength function of the $1h_{9/2}$ and $1i_{13/2}$ subbarrier one-neutron states in the ^{113}Sn nucleus is qualitatively consistent with the spectrum of the reaction ^{112}Sn$(\alpha,^3$He$)^{113}$Sn (the unrenormalized strength function),[31] which also has two maxima with energies close to the energies of the maxima of the calculated dependence.

5. As an example illustrating the nature of the change of the strength function in going from "hard" to "soft" nuclei, in Fig. 9 we give the results of calculations of the dependence $S_\lambda(E_x)$ of the $(1g_{9/12})_p^{-1}$ state in the nuclei $^{141-147}$Pr. The nature of the isotopic dependence of the strength function is a consequence of the coupling of the quasiparticle to multiphonon configurations.

6. In hard spherical nuclei the results of the MVCCM are close to the corresponding results of the "few-phonon" approaches, and for soft nuclei they differ qualitatively (see Figs. 7 and 8).

7. The closeness of the results of calculations of the strength function of the $(1g_{9/2})_n^{-1}$ state in the nucleus ^{123}Te, carried out using the MVCCM in the two-phonon approximation, to the corresponding results of the QPM (Ref. 34) shows that the structure of this strength function found in the QPM is a consequence of the use of a few-

TABLE III. Results of calculations of the parameters of the energy dependences of the one-quasiparticle strength functions. Only the quasiparticle interaction with 3^- phonons is taken into account explicitly.

Nucleus	State	Energy range, MeV	E_x, MeV		Γ, MeV		S_λ	
			Theory	Experiment	Theory	Experiment	Theory	Experiment
^{89}Zr	$(1f_{5/2})_n^{-1} + (1f_{7/2})_n^{-1}$	0–20	7.01	6	5.61	4.1	0.92	0.63
^{207}Pb	$(1h_{11/2})_n^{-1}$	6.7–10.5	8.78	8.5	1.51	3.7	0.81	-
	$(1g_{9/2})_n^{-1}$	10.5–17	14.9	14	2.44	5.1	0.97	0.92
^{209}Bi	$(1i_{11/2})_p$	4–13	8.13	7.8	2.83	4.8	0.98	0.72
	$(1j_{15/2})_p$	4–13	8.19	7.4	4.04	5.4	0.92	0.77

TABLE IV. Results of calculations of the parameters of the energy dependences of the one-quasiparticle strength functions. Only the quasiparticle interaction with 2^+ phonons is taken into account explicitly.

Nucleus	State	E_m, MeV Theory	E_m, MeV Experiment	Γ, MeV Theory	Γ, MeV Experiment
^{113}Sn	$(1h_{9/2})_n+(1i_{13/2})_n$	6.82	-	1.26	-
		8.81	-	4.48	-
^{117}Sb	$(1h_{9/2})_p+(1i_{13/2})_p$	10.3	10.8	5.60	5.5
^{145}Eu	$(2f_{7/2})_p+(1h_{9/2})_p$	5.48	5.9	1.25	1.2
	$+(1i_{13/2})_p$	7.09	7.6	4.90	4.0

phonon basis (see Fig. 7).

5. MVCCM ANALYSIS OF THE NEUTRON STRENGTH FUNCTIONS AND RADII OF NUCLEON ELASTIC SCATTERING ON SPHERICAL NUCLEI

Fundamental relations. Qualitative analysis

Among the observable consequences of the relaxation of the one-particle degree of freedom in the nucleus are the neutron strength functions (NSF) of compound resonances. The problem of the quantitative interpretation of the NSF is a problem of the continuous spectrum. In the case of nonoverlapping compound resonances with fixed values of the angular momentum and parity, the neutron strength function S_n is related to the corresponding element of the averaged neutron–nucleus scattering matrix according to Eq. (32). In order to eliminate the trivial energy dependence in $S_n(\varepsilon)$ due to the penetrability of the potential barrier for neutrons, it is convenient instead to analyze the reduced strength function $S_j^l(kR \ll 1)$:

$$S_j^l=(kR)^{-2l}\varepsilon^{-1/2}(S_n)_j^l. \tag{99}$$

Here ε is expressed in eV.

The reduced s- and p-NSF $S^0\equiv S^0_{1/2}$, $S^1_{1/2,3/2}$, and $S^1\equiv\frac{1}{3}S^1_{1/2}+\frac{2}{3}S^1_{3/2}$ are of practical interest. In the MVCCM the difference of the diagonal element of the averaged scattering matrix from the optical-model value is determined by the irreducible self-energy part ΔH^p_{jl} according to (53), (54), and (71). From these relations it follows that in the final analysis the NSF, like the one-quasiparticle strength functions, are determined by the radial Green functions $g_{jl}(rr',\varepsilon)$, which must be found by solving the system of nonlinear integral equations (52), taking into account (71). Therefore, Eqs. (30), (32), (52)–(54), (71), and (99) are the formal solution of the problem of calculating the reduced neutron strength functions in the MVCCM.

As shown by the optical-model analysis[1,17,35] (see also Sec. 2), the A dependence of the reduced s and p NSF has maxima corresponding to s and p shape resonances. For example, the maximum of the dependence $S^0(A)$ corresponds to the passage through zero of the neutron $3s$ level ($A\sim55$), and the maxima of the dependences $S^1_{3/2}(A)$ and $S^1_{1/2}(A)$ correspond to the $3p_{3/2}$ and $3p_{1/2}$ levels ($A\sim100$ and $A\sim110$, respectively). As a rule, for nuclei in the vicinity of these shape resonances the coupling of the corresponding one-particle configurations to 2^+ phonons is strong, and therefore the MVCCM should be used for the quantitative analysis of S^0 and $S_{1/2,3/2}$ in this case. In this regard we note that there is no "self-interaction" effect for the $s_{1/2}$ and $p_{1/2}$ configurations. In these cases there can be strong coupling between states close in energy: $3s$ and $2d$, $3p$ and $2f$, $3p_{1/2}$ and $3p_{3/2}$. In going away from the shape resonance the effective quasiparticle – 2^+-phonon coupling decreases [this can be seen, for example, from Eq. (66)], and the value of the NSF becomes proportional to the strength of the imaginary part of the optical potential w. However, owing to the relatively large value of the dynamical quadrupole deformation parameter, it is possible for "local" maxima to appear in the dependence $S(A)$. This situation is realized in the case of the s NSF near $A\sim75$ and $A\sim105$ (Ref. 36), and for the p NSF near $A\sim75$ (Ref. 37).

Results of the NSF calculations

The method of solving the system of nonlinear integral equations for the radial Green functions $g_{jl}(rr',\varepsilon)$ is described in Sec. 4. Numerical calculations of $S^{0,1}$ have shown that the contact-interaction approximation (63) is not quantitatively accurate: the use of a realistic interaction in the case of strong quasiparticle – 2^+-phonon coupling can change the calculated values of the s and p NSF by a factor of 1.5–2. This is not surprising, since the wave functions of the $3s$ and $3p$ states each have two nodes and therefore vary noticeably over a distance of the order of the

TABLE V. Values of the parameters ν^0 and η for a number of one-quasiparticle states of spherical nuclei.

Nucleus	State	ν^0	η
^{89}Zr	$(1f_{5/2})_n$	0.05	0.91
	$(1f_{7/12})_n$	0.09	0.95
^{109}Pd	$(2p_{1/2})_n$	5.03	1.14
	$(2p_{3/2})_n$	5.22	1.21
	$(1g_{9/2})_n$	6.19	0.99
^{111}Sn	$(1g_{9/2})_n$	0.69	1.02
^{113}Sn	$(1g_{9/2})_n$	0.76	0.97
	$(1i_{13/2})_n$	1.68	0.77
^{115}Sn	$(2p_{1/2})_n$	0.69	0.82
	$(2p_{3/2})_n$	0.71	0.77
	$(1g_{9/2})_n$	0.86	0.94
^{119}Sn	$(1g_{9/2})_n$	0.93	0.87
^{121}Te	$(1g_{9/2})_n$	2.74	1.07
^{123}Te	$(1g_{9/2})_n$	2.73	1.03
^{129}Te	$(1g_{9/2})_n$	1.50	0.93
^{143}Sm	$(1g_{9/2})_n$	0.59	0.97
^{145}Eu	$(2f_{7/2})_p$	1.13	0.72
	$(1h_{9/2})_p$	0.87	1.39
	$(1h_{13/2})_p$	1.33	0.94
^{207}Pb	$(1h_{11/2})_n$	0.43	1.04
	$(1g_{9/2})_n$	0.24	0.80
^{209}Bi	$(1i_{11/2})_p$	0.08	0.74
	$(1j_{15/2})_p$	0.11	0.83

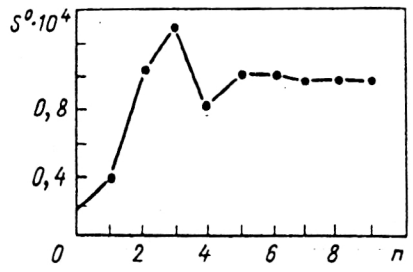


FIG. 16. Results of calculations of the dependence of the S^0 NSF in the nucleus ^{105}Ru on the number of phonons taken into account.

diffusivity of the shell-model potential near $r \sim R$. The effective number of phonons n_{eff} contributing to NSF formation can be found by using the method described in Sec. 3 for solving the system of equations for the radial Green functions with the replacement $g_{jl}(rr', \varepsilon - n\omega_L) \to g_{jl}^{\text{opt}}(rr', \varepsilon - n\omega_L)$. As an example of finding n_{eff}, in Fig. 16 we give the results of calculating the dependence $S^0(n)$ for the soft nucleus ^{105}Ru. For the value $w = 1.5$ MeV used in the calculation and the known accuracy of the calculation (10%), it turned out that $n_{\text{eff}} = 5$. A criterion for strong quasiparticle–2^+-phonon coupling is obviously the condition $n_{\text{eff}} \gg 1$.

The strength of the imaginary part of the optical potential is the only independently variable parameter of the model. The other parameters must be chosen as in the calculations of the one-quasiparticle strength functions. Qualitative analysis of the fundamental relations of the MVCCM (see Sec. 3) and the CCM (see Sec. 2) shows that in the case of strong quasiparticle–2^+-phonon coupling the dependence of the NSF on w should be weak up to rather large w in the MVCCM calculations and strong in the CCM calculations. These conclusions are confirmed by the results of numerical calculations. As an example, in Fig. 17 we show the dependences $S^0(w)$ obtained in the MVCCM and the CCM for the soft nucleus ^{77}Se. Therefore, near the maxima we can expect weak dependence of the NSF on w, while near the minimum, where the channel coupling is hardly manifested at all, the NSF is proportional to w. It is thus possible to choose the latter from, for example, the experimental values of S^0 near the minimum. The value of w corresponding to agreement between the

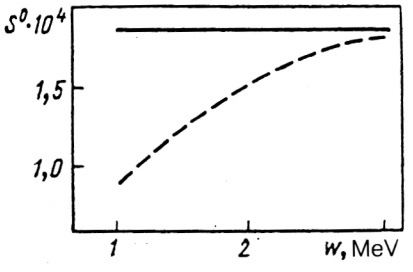


FIG. 17. Results of calculations of the dependence of the S^0 NSF in the nucleus ^{77}Se on the strength of the imaginary part of the optical potential w, calculated using the MVCCM (solid line) and the CCM (dashed line).

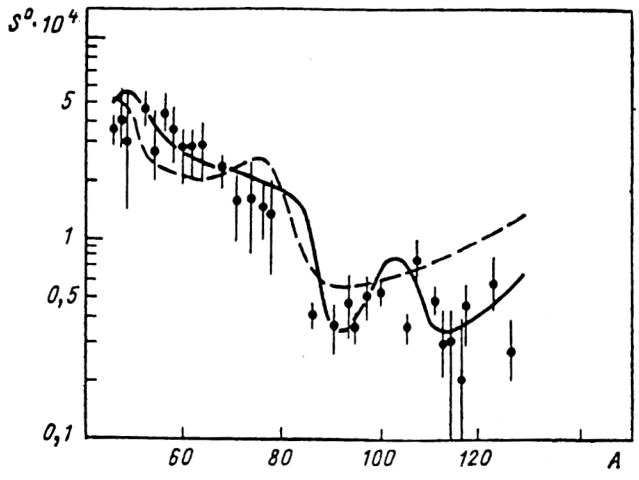


FIG. 18. Results of calculations of the A dependence of the S^0 NSF compared with the experimental data of Ref. 38.

experimental and calculated values of S^0 near the minimum in the average over A turned out to be 1.5 MeV, which is half the value of w obtained using the CCM to simultaneously describe both the maximum and the minimum of the dependence $S^0(A)$ (Ref. 37). In Figs. 18–20 together with the experimental data we give the results of calculations of $S^0(A)$, $S^1(A)$, and $S^1_{1/2,3/2}(A)$ for $w = 1.5$ MeV carried out using the MVCCM. The experimental data for $S^{0,1}$ are taken from Ref. 38 and those for $S^1_{1/2,3/2}$ are from Ref. 39. In Figs. 18 and 19 we also give the results of calculations of $S^{0,1}(A)$ carried out using the CCM with $w = 3.0$ MeV (Ref. 37). It follows from the data shown in Figs. 18–20 that the MVCCM leads to a satisfactory description of the experimental data on the reduced s and p NSF using the same parameters of the imaginary part of the optical potential. At the minimum of the A dependence of S^0 ($A \sim 110$–120) this description is obtained for the average over A. The use of the same parameters of the imaginary part of the optical potential is an advantage that

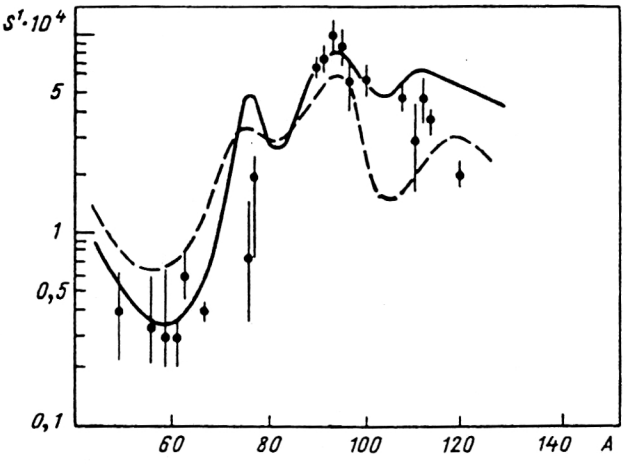


FIG. 19. The same as in Fig. 18 for the S^1 NSF.

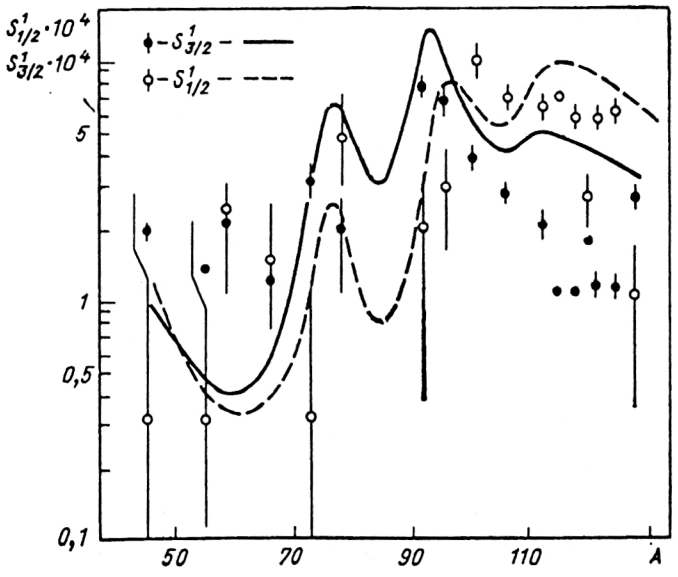


FIG. 20. The same as in Fig. 18 for the $S^1_{1/2}$ and $S^1_{3/2}$ NSF. The experimental data are taken from Ref. 39.

our method has over the CCM, which even for averaging over A cannot satisfactorily describe the experimental data on $S^0(A)$ near the maximum and the minimum using the same value of w (Ref. 37).

In order to explain the isotopic behavior of the NSF near the minimum of the A dependence it is necessary to take into account shell effects in the formation of the value of w and a possible energy dependence of this quantity.[40] The analysis of these effects lies outside the scope of the present review. In relation to this we note that here the description of the NSF maximum is changed only slightly.

The neutron elastic-scattering radii

Among the parameters determining the averaged cross section for low-energy scattering of neutrons by nuclei, in addition to the NSF there are the elastic-scattering radii R^l_j. Just as the reduced neutron strength functions S^l_j (99) are determined by the imaginary part of the phase of the corresponding diagonal element of the averaged scattering matrix $\langle S_{nn}\rangle$ [see (32)], the elastic-scattering radii are determined by the real part of this phase:

$$\xi_{jl} = -\{(kR)^{2l}/[(2l+1)!!(2l-1)!!]\}kR^l_j. \qquad (100)$$

The experimental data on the R^l_j are extracted from analysis of the differential cross section for low-energy neutron elastic-scattering.[38,39] In relation to this the s and p elastic-scattering radii are of practical interest:

$$R^0 = R^0_{1/2};\ R^1 = \frac{1}{3}R^1_{1/2} + \frac{2}{3}R^1_{3/2}. \qquad (101)$$

In the vicinity of the corresponding shape resonance the A dependence of the elastic-scattering radius undergoes a strong variation relative to the dependence at some effective radius (close to the nuclear radius), with the amplitude and interval ΔA on which this variation occurs determined by the damping of the corresponding one-particle resonance. In particular, the values of $R^{0,1}$ can become negative. The MVCCM calculation of the elastic-scattering radii using (30), (32), and (100) is based on the same expressions (52)–(54) and (71) used above for the quantitative analysis of the NSF. In the calculations of $R^{0,1}$, the results of which are given in Figs. 21 and 22, we used the same methods and the same phenomenological parameters as in the CCM calculations. In these figures we also give the experimental values from Ref. 38 (for R^0) and 39 (for R^1) and (in Fig. 21) the results of the calculations of R^0 using the "two-parameter" CCM approach, in which the coupling of the one-quasiparticle state to multiphonon configurations is effectively taken into account in the average over A using a new phenomenological parameter.[36]

6. BROADENING OF THE DIPOLE GIANT RESONANCE IN SOFT SPHERICAL NUCLEI

An important manifestation of the strong quasiparticle–2^+-phonon coupling should also be expected in the damping of multipole giant resonances (GR) in soft spherical nuclei. This is indicated, in particular, by the strong A dependence of the total width of the $E1$ GR: the $E1$ GR width varies from 4 MeV in hard nuclei to 9 MeV

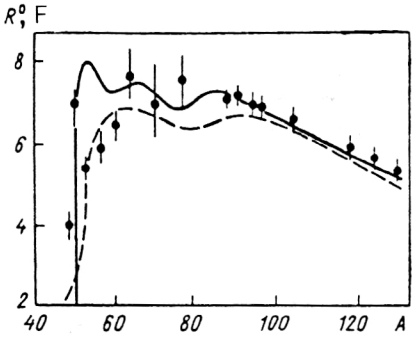


FIG. 21. Results of calculations of the A dependence of R^0 (solid line) compared with the experimental data of Ref. 38 and with the results of the "two-parameter" CCM of Ref. 36 (dashed line).

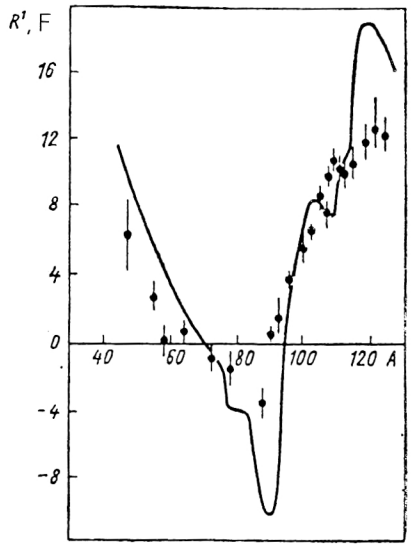


FIG. 22. Results of calculations of the A dependence of R^1 compared with the experimental data of Ref. 39.

in soft ones.[41] A similar situation also occurs for the isoscalar $E2$ GR (Ref. 42).

The first attempt to theoretically study the broadening of the dipole GR was made in Ref. 43, where the $E1$ GR was viewed as a quasiparticle interacting only with 2^+ phonons, with the basis of the corresponding diagonalization problem including multiphonon configurations (up to 14 2^+ phonons). By solving this diagonalization problem, the authors of Ref. 43 determined the $E1$ GR strength distribution and its total width $\Gamma \equiv 2.35\sigma$, where σ^2 is the variance of the strength distribution. The results of the calculations of the width Γ correctly reproduce the observed A dependence of the total width of the $E1$ GR, lowering the absolute values of the width in soft nuclei by 1.5–2 MeV. Naturally, in hard nuclei the calculated width is practically equal to zero. These deficiencies of the approach of Ref. 43 are related to the fact that it does not include other important sources of formation of the total width of the giant resonance: the energy spread of the strength of collective states of the particle–hole type, $\Delta\Gamma$ (in the average over A, $\Delta\Gamma \sim 2$ MeV; Ref. 44), and the coupling of these states to multiparticle configurations. The listed relaxation modes form the "proper" width $\Gamma^{(0)}$, which coincides with the width of the $E1$ GR in hard nuclei, and the width calculated in Ref. 43 is naturally termed the phonon width of the giant resonance, Γ_{ph}.

It is possible to take a more systematic approach to the description of the total width of the GR in which the GR is treated microscopically using the shell model, and the damping of the quasiparticles forming it is described using the MVCCM. In hard nuclei this approach has been realized and essentially corresponds to the optical shell model (OSM).[45] The realization of this approach in soft nuclei, which requires the inclusion of an additional damping interaction between quasiparticles owing to 2^+-phonon exchange, is a very difficult problem, which so far has not been solved. However, it is possible to formulate an approximate method of calculating the strength function of the dipole GR in which the "proper" width of the resonance is taken into account in addition to the phonon width. The conditions $\omega_G > \omega$, $\Gamma > \Delta\Gamma$, $\Gamma > \omega$, and $\Delta\omega > \varkappa$, where ω_G is the $E1$ GR energy, ω is the 2^+-phonon energy, $\varkappa$ is the constant describing the coupling of the $E1$ GR to the 2^+ phonon, and $\Delta\omega$ is the shift of the energy of the maximum of the $E1$ GR relative to the energy of the maximum of the $E1$ GR in the independent-quasiparticle model, allow the $E1$ GR to be treated as a quasiparticle with proper width interacting with a 2^+ phonon. The constant describing this quasiparticle–phonon coupling can be parametrized using simple qualitative considerations:

$$\varkappa = C\beta\omega_G/\sqrt{3}, \tag{102}$$

where β is the dynamical quadrupole deformation parameter of the nucleus and the coefficient $C \sim 1$ can either be taken to be a phenomenological parameter, or can be found by model calculations. We note that so far it has not proved possible to calculate this coefficient using the results of RPA calculations of the $E1$ GR strength distribution with a sufficiently complete particle–hole basis.

This approximate approach to describing the $E1$ GR strength function allows direct use of the fundamental equation of the MVCCM in the form (74) for the Green function of the $E1$ GR quasiparticle:

$$G(\varepsilon) = G_0(\varepsilon) + \varkappa^2 G_0(\varepsilon) G(\varepsilon - \omega) G(\varepsilon), \tag{103}$$

where $G_0(\varepsilon) = (\varepsilon - \omega_G + i\Gamma^{(0)})^{-1}$. The zeroth approximation in (103) is chosen such that in the limit $\varkappa \to 0$ the strength function of the $E1$ GR $S(\varepsilon) = -(1/\pi)\,\mathrm{Im}\,G(\varepsilon)$ is described by a Breit–Wigner curve with width $\Gamma^{(0)}$. The method of solving Eq. (103) is described in detail in Sec. 3. We note that according to Eq. (103) in the limit $\Gamma \gg \omega$, when $G(\varepsilon - \omega) = G(\varepsilon)$, this equation can be solved analytically. The approximation of the calculated strength function by a Breit–Wigner curve allows the calculated value of the total width Γ of the $E1$ GR to be found.

In the calculations of the absolute value and the A dependence of the width Γ, the results of which are shown in Fig. 23, the values of the parameters β and ω were taken from Ref. 25, and ω_G and $\Gamma^{(0)}$ are from Refs. 41 and 46, where the proper width of the $E1$ GR was identified with the width of the $E1$ GR in the hard nucleus closest to the one studied. The coefficient $C = 0.85$ in (102) used in the calculation of the widths is determined such that this method reproduces the observed width of the $E1$ GR in isotopes of Ge and Se.[46] We note that the authors of Ref. 43 used the similar value $C = 0.77$ calculated in the hydrodynamical model. For comparison, in Fig. 23 we give the values of the phonon width Γ_{ph} calculated from the solution of Eq. (103) in the limit $\Gamma^{(0)} \to 0$. It follows from the data given in Ref. 23 that in the simplified version of the MVCCM taking into account the dominant relaxation modes it is possible to obtain a satisfactory description of both the A dependence and the absolute values of the total width of the $E1$ GR in spherical nuclei of intermediate weight.

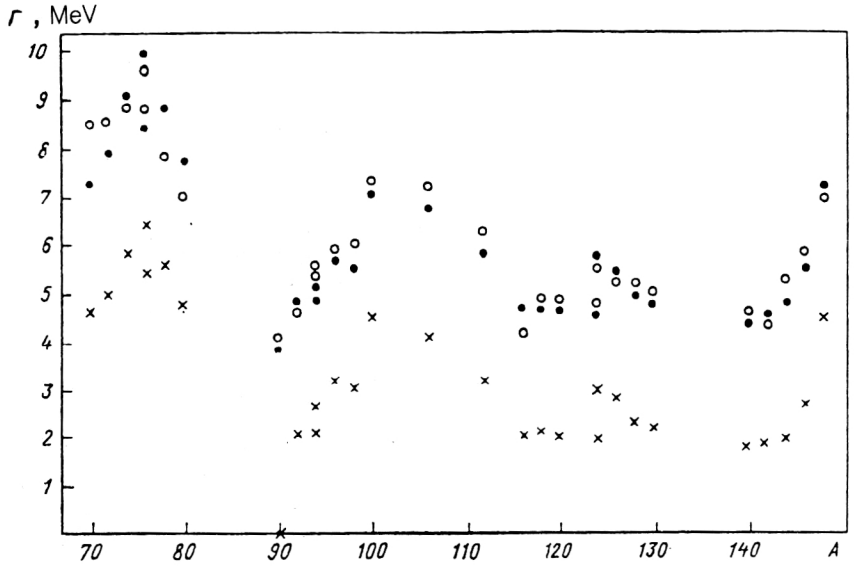


FIG. 23. Results of calculations of the A dependence of the total width of the $E1$ GR (points) compared with the experimental data of Refs. 41, 42, and 46 (circles). The crosses show the values of the phonon width of the $E1$ GR.

Further progress in the quantitative interpretation of the strength functions of GR in soft nuclei requires the microscopic calculation of the constant $\varkappa$ and, eventually, the realization of the approach mentioned above corresponding to the synthesis of the OSM and the MVCCM.

7. NUCLEON DECAY OF SUBBARRIER ONE-PARTICLE STATES IN SOFT SPHERICAL NUCLEI

The next step in the study of the relaxation of one-quasiparticle states with high excitation energy is the analysis of their specific decay channels. Here we consider the nucleon decay of subbarrier one-particle states in soft spherical nuclei. We restrict ourselves to states with large angular momentum and parity opposite to that of the shell. In this case, where 2^+ phonons can be treated as scalar particles,[5] it is possible to obtain simple analytic expressions for the reduced differential probabilities for the nucleon decay of subbarrier states to the ground (0^+) and first excited (2^+) states of the daughter nucleus.

Let $\lambda \equiv nlj$ be the quantum numbers of the subbarrier state with parity opposite to that of the shell. Since the coupling of this state to other one-particle states owing to 2^+-phonon exchange can be neglected (these states belong to neighboring shells) and $j \gg 2$, the interaction between the λ particle and the 2^+ phonon can be viewed as the interaction of this particle with a scalar phonon.[5] In this approximation the differential probabilities for nucleon decay of the state λ to the ground and first excited states of the daughter nucleus are determined by the equations

$$d\Gamma_\lambda^{(0,1)}/dE_x = S_\lambda^{(0,1)}(E_x)\Gamma_\lambda(\varepsilon_k), \tag{104}$$

where $S_\lambda^{(0)}$ is the one-particle strength function, Γ_λ is the one-particle width for decay into the continuum, and ε_k is the nucleon kinetic energy. The quantity $S_\lambda^{(1)}(E_x)$ in (104) can be termed the particle–phonon strength function. The particle–hole strength function of the one-particle state in question can be defined as

$$S_\lambda^{(1)} = \langle |\langle s|b^+ a_\lambda^+ |0\rangle^2\rangle \rho_s, \tag{105}$$

where a_λ^+ and b^+ are the particle and (scalar) phonon creation operators. The remaining notation is the same as in Sec. 1.

Since in "soft" nuclei the fragmentation of the one-particle state is mainly determined by the strength of the particle–2^+-phonon coupling (see Sec. 4), for the qualitative analysis of the strength functions $S_\lambda^{(0,1)}$ let us consider the problem of the interaction of the particle λ with only scalar phonons. This problem corresponds to the Hamiltonian (79), and the strength functions $S_\lambda^{(0,1)}$ are determined by the weights of the one-particle w_n^s (78) and particle–phonon w_n^{s1} (80) components of the exact wave functions of the Hamiltonian (79): $S_\lambda^{(0)} = \bar{w}_n^s/\omega$, $S_\lambda^{(1)} = \bar{w}_n^{s1}/\omega$, where the bar denotes averaging over the energy interval I' such that $\varkappa \gg I' \gg \omega$ (see Sec. 3). It follows from Eqs. (78) and (80) that in the case of strong particle–phonon coupling near the maximum of the one-particle resonance the strength functions $S_\lambda^{(0,1)}$ are determined by the expressions

$$S_\lambda^{(0,1)} = \frac{1}{\sqrt{2}\varkappa} |\Psi_{0,1}(\xi)|^2, \tag{106}$$

where $\sqrt{2}\varkappa\xi = \varepsilon - \varepsilon_\lambda$ and $\Psi_{0,1}(\xi)$ are the wave functions of the ground and first excited states of the linear harmonic oscillator. Therefore, the strength functions $S_\lambda^{(0)}$ and $S_\lambda^{(1)}$ calculated for this problem, respectively, have one and two maxima. The energy interval between the maxima of the strength function $S_\lambda^{(1)}$ (equal to $2\sqrt{2}\varkappa$) is comparable with the width of the maximum of the strength function $S_\lambda^{(0)}$ (equal to $2.35\varkappa$).

We use the Green-function method for the quantitative analysis of the strength functions $S_\lambda^{(0,1)}$. In addition to the one-particle Green function $G_\lambda(\tau)$ [cf. (1)], we introduce the particle–phonon Green function $F_\lambda(\tau)$:

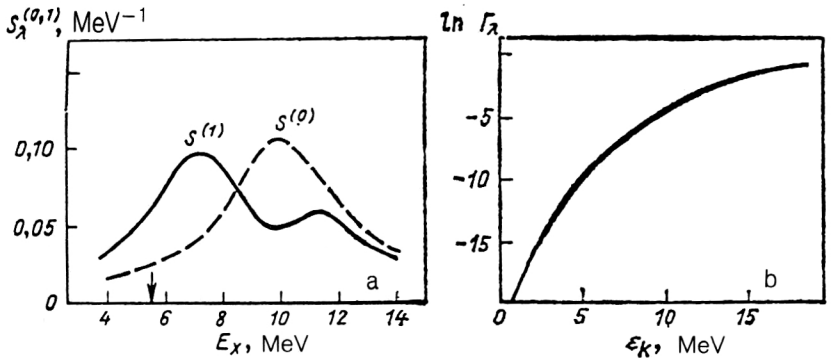


FIG. 24. Results of calculations of the strength functions $S_\lambda^{(0,1)}(E_x)$ (a) and the one-particle neutron width Γ_λ (b) for the $(1i_{13/2})_n$ state in the nucleus ^{101}Mo. The arrow indicates the neutron binding energy. Γ is expressed in MeV.

$$G_k(\tau=t-t')=-\mathrm{i}\langle 0|Ta_\lambda(t)a_\lambda^+(t)'|0\rangle;$$

$$F_\lambda(\tau=t-t')=-\mathrm{i}\langle 0|Ta_\lambda(t)b(t)a_\lambda^+(t')b^+(t')|0\rangle. \quad (107)$$

Here $a_\lambda(t)$, $a_\lambda^+(t)$ and $b(t)$, $b^+(t)$ are Heisenberg operators. According to the spectral expansion, the Fourier transforms of the Green functions (107), $G_\lambda(\varepsilon)$ and $F_\lambda(\varepsilon)$, determine the strength functions $S_\lambda^{(0,1)}(\varepsilon)$ (105) [cf. (3) and (6)]:

$$S_\lambda^{(0)}(\varepsilon)=-\frac{1}{\pi}\operatorname{Im} g_\lambda(\varepsilon);\ S_\lambda^{(1)}=-\frac{1}{\pi}\operatorname{Im} f_\lambda(\varepsilon), \quad (108)$$

where $g_\lambda(\varepsilon)=\langle G_\lambda(\varepsilon)\rangle$ and $f_\lambda(\varepsilon)=\langle F_\lambda(\varepsilon)\rangle$.

The problem considered above of the interaction of a particle with scalar phonons corresponds to the Green functions $G_\lambda(\varepsilon)$ and $F_\lambda(\varepsilon)$, respectively, satisfying Eqs. (77) and

$$F_\lambda(\varepsilon)=G_\lambda(\varepsilon-\omega)+\omega^{-1}\varkappa^2[G_\lambda^{(0)}(\varepsilon-\omega)F_\lambda(\varepsilon-\omega) - G_\lambda^{(0)}(\varepsilon)F_\lambda(\varepsilon)], \quad (109)$$

which follow from (79) and (107). In (109), $G_\lambda^{(0)}(\varepsilon)=(\varepsilon-\varepsilon_\lambda)^{-1}$. In this approximation Eqs. (77) and (109) take into account the coupling of the particle λ only to configurations containing 2^+ phonons. According to the conclusions of Sec. 2, the coupling of the particle λ to other ("noncollective") multiparticle configurations in soft nuclei can be taken into account in terms of an imaginary part of the optical potential with smooth energy dependence $w(r,E_x=\varepsilon-\mu)$. This means that the averaged Green functions $g_\lambda(\varepsilon)$ and $f_\lambda(\varepsilon)$ satisfy equations of the form (77) and (109) with the replacement $G_\lambda^{(0)}(\varepsilon)\to g_\lambda^{(0)}(\varepsilon)=(\varepsilon-\varepsilon_\lambda+\mathrm{i}w_\lambda)^{-1}$ [cf. (43)]. Here the quantity $w_\lambda(E_x)$ is determined by Eq. (45):

$$w_\lambda(E_x)=\int w(r,E_x)\chi_\lambda^{(0)2}(r)dr. \quad (110)$$

After this substitution Eqs. (77) and (109) can, with accuracy $\omega/\sqrt{\varkappa^2+w_\lambda^2}$, respectively, be transformed into the differential equations (84) and

$$f_\lambda(\varepsilon)=g_\lambda(\varepsilon)-\varkappa^2 d[g_\lambda^{(0)}(\varepsilon)f_\lambda(\varepsilon)]/d\varepsilon. \quad (111)$$

In the approximation $dw_\lambda/d\varepsilon \ll 1$ the solution of Eq. (84) is determined by (86), in which $I'\to w_\lambda$, and the solution of Eq. (111) is determined by

$$\varkappa f_\lambda(\varepsilon)=(x+\mathrm{i}\bar{w}_\lambda)^2\varkappa g_\lambda(\varepsilon)-(x+\mathrm{i}\bar{w}_\lambda);$$

$$x=(\varepsilon-\varepsilon_\lambda)/\varkappa;\quad \bar{w}=w_\lambda/\varkappa. \quad (112)$$

Naturally, near the one-particle resonance in the limit $w_\lambda\to 0$ we arrive at (106) on the basis of (86), (108), and (112).

To calculate the strength functions $S_\lambda^{(0,1)}$ according to (86), (108), and (112) it is necessary to know the parameters ε_λ, $\varkappa$, and w_λ. The energies ε_λ and radial wave functions $\chi_\lambda(r)$ are identified with the energies and wave functions of the corresponding quasistationary states in the one-particle potential (88)–(90) and are found by numerical calculation. The coupling constant $\varkappa$ is calculated using the expression following (66). The quantity w_λ is calculated as in (110). According to (106), in soft nuclei analysis of the irregularities in the energy dependence of the strength function $S_\lambda^{(1)}(E_x)$ requires the study of a fairly large energy interval. Therefore, in the calculations it is necessary to include the energy dependence of the strength of the imaginary part of the optical potential $W(E_x)$. This is taken to be linearly dependent on the excitation energy: $W(E_x)=\alpha E_x$ (Ref. 15), $\alpha=0.15$. For excitation energies $E_x\sim 5$–7 MeV this choice of α leads to values of w_λ which coincide with those used in Secs. 3–5.

In Figs. 24a and 25a we show the results of calculations of the strength functions $S_\lambda^{(0,1)}(E_x)$ using Eqs. (86), (108), and (112). To estimate the absolute value of the nucleon decay probability (104), in Figs. 24b and 25b we give the energy dependences of the corresponding one-particle nucleon widths $\Gamma_\lambda(\varepsilon_k)$ calculated for a potential well with a sharp edge.[15]

It follows from this discussion that in soft spherical nuclei the strength functions of one-particle states with large angular momentum and parity opposite to that of the shell, $S_\lambda^{(0)}(E_x)$ and $S_\lambda^{(1)}(E_x)$, respectively, have one and two maxima. To discover this effect experimentally it is interesting to study reactions with nucleon transfer to an even nucleus with subsequent nucleon decay of the subbarrier one-particle state into the ground and first 2^+ states of the daughter nucleus.

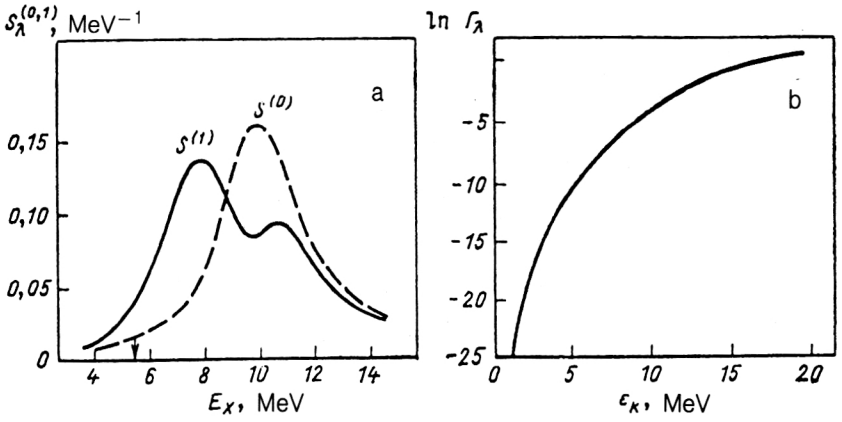


FIG. 25. The same as in Fig. 24 for the $(1j_{15/2})_n$ state in the nucleus ^{147}Nd. Γ is expressed in MeV.

CONCLUSION

The preceding discussion can be summarized as follows.

1. We have formulated a simple version of the multiphonon variant of the coupled-channel method in which it is possible to take into account the coupling of quasiparticles to multiphonon configurations. In practical realizations of the method one uses the same phenomenological parameters as in the "few-phonon" versions of the CCM. These parameters are found from independent data. This feature together with the relative simplicity of the realization makes the proposed method an effective tool for the theoretical analysis of the observed consequences of the relaxation of the one-quasiparticle degree of freedom in soft nuclei for excitation energies which are not too high.

2. We have demonstrated the possibilities of this method in the quantitative interpretation of the one-quasiparticle and neutron strength functions, the radii of low-energy neutron elastic scattering, and the total width of the $E1$ GR in soft nuclei. The results of the calculations of these quantities using the MVCCM for a large number of nuclei are in satisfactory agreement with the corresponding experimental data. Objects for further analysis using the MVCCM are the parameters of the cross sections for elastic and inelastic (with excitation of the first 2^+ state) cross sections for low-energy neutron scattering by spherical nuclei.

3. The relative simplicity of the formulation and realization of the method are the consequence of the use of a number of approximations, without which one could attempt to interpret the detailed structure of the strength functions. These approximations are the neglect of anharmonic effects (including the effect of the Pauli principle on multiphonon states) and of the renormalization of the quasiparticle–phonon vertex. The quantitative analysis of these approximations and the inclusion of shell effects in the formation of the imaginary part of the optical potential could be subjects for future study. The formulation and realization of a more systematic method of analyzing the strength functions of the $E1$ GR in soft nuclei is also quite difficult. Such an approach should include not only the coupling of each of the $E1$ GR quasiparticles formed to multiphonon and multiparticle configurations (as was done in the MVCCM for the one-quasiparticle strength functions), but also the additional retarded interaction of these quasiparticles owing to 2^+-phonon exchange.

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